Altruism and the Transfer Paradox

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Abstract

The paper examines whether altruism causes the transfer paradox in the model with two countries and two goods. Unlike the existing literature on the subject in which the people in the donor country maximize their own utility and do not care about the people in the recipient country, we analyze a situation in which the consumers of the donor and recipient countries have altruistic utility. We demonstrate that if the Walrasian stability condition is satisfied in the general equilibrium, the transfer paradox can never take place irrespective of the definition of utility. The result suggests that the motivation for transfer cannot be explained by the donor’s enrichment because it is not caused by the introduction of altruism into the model.

Keywords: Altruism; Transfer paradox; Walrasian stability

JEL classifications: F11, F35

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1 Introduction

Since Keynes vs. Ohlin debated over the German war reparations after World War I — a well-known classical example of debates on international transfers of income — the transfer problem has sparked a lot of public interest and has raised various economic issues in the international trade theory.\(^1\) An income transfer may improve or worsen the terms of trade of both the transferer and transferee. If the secondary effect of the income transfer on the terms of trade exceeds its direct effect, the welfare of the transferer may improve and/or the welfare of the transferee may worsen against the will of the former. Such a situation is called the “transfer paradox.” Leontief (1936) presented an example of the fact that the change of terms of trade resulting from a transfer causes the transfer paradox for the donor through one of the classical articles that dealt with the transfer paradox in a two-commodity world involving two countries. On the contrary, using the notion of Walrasian stability, Samuelson (1952, 1954) showed that if the general equilibrium is stable, the transfer paradox cannot arise under free trade and as a result, the example of Leontief (1936) is excluded under the stability assumption.

In order for the transfer paradox to take place in a stable equilibrium, some presumptions of Samuelson (1952) need to be modified. The existing literature on the subject has explained why the transfer paradox takes place by mainly extending Samuelson’s model in the following two directions. The first extension is the introduction of the third country into the model. In the model with three countries and two goods, Bhagwati et al. (1983) show that there is a possibility of the paradoxes of enriched donor and immiserized recipient arising when there is an outside country in a multilateral world. Second, when free trade is hindered by distortions such as trade barriers, the transfer paradox takes place in a two-country model. Bhagwati et al. (1985) demonstrate that when there is exogenous distortion by trade barriers such as tariff and subsidy, a transfer may paradoxically enrich the donor and immiserize the recipient. They also show that if the transfer induces endogenous distortion such as lobbying and rent-seeking, there exists the

\(^1\) Regarding the Keynes vs. Ohlin controversy, see Keynes (1929) and Ohlin (1929). As an example of a concise survey on the transfer problem, see Brakman and Marrewijk (1999, Ch.2).
possibility of paradoxes of donor and recipient arising. The existing literature has analyzed a variety of distortions. Administrative cost of transfer, sticky wage and unemployment, and transfer of production factors are some examples.²

In recent years, several articles have attempted to explain the transfer paradox by introducing altruism into the donor’s utility in the model. If a donor has altruistic utility, the transfer raises the recipient’s utility and the increase in the recipient’s utility may in turn raise the donor’s utility. If this result is correct, the introduction of altruism into the donor’s utility explains the donor’s enrichment. As a result, altruism presents the reason behind the donor’s voluntary contribution of foreign aid. Kemp and Shimomura (2002) explore the model of voluntary unrequited transfer and show that altruism might be the motivating factor behind the donor’s transfer. Contrary to altruism, in the setting in which the welfare of each country is negatively influenced, Kemp and Shimomura (2003) demonstrate that the donor might benefit at the expense of the recipient. Lahiri and Raimondos-Møller (1999) develop a model wherein altruism is the motive for the donor giving aid and trade is distorted by tariffs or quotas. They show that if the donor is sufficiently altruistic, transfer is strictly Pareto improving. In other words, they conclude that the donor’s altruism enriches the donor itself. Takarada and Tawada (2003) explore the model wherein the donor has altruistic utility and the donor’s government gives aid for attaining political objectives. They demonstrate that it is always optimal for the recipient to accept the transfer and if the donor takes care of the recipient’s welfare sufficiently, the donor’s enrichment takes place.

However, most of the existing literature has explained the transfer paradox by combining altruism with other distortions such as tariffs or political motives into the model. In the present article, we examine whether altruism itself causes the transfer paradox in a simple two-commodity world with two countries when there is no other distortion. By applying the traditional argument on the stability of the general equilibrium, we demonstrate that even if

² The distortions that cause the transfer paradox have been comprehensively analyzed by Brakman and Marrewijk (1999).
the donor and/or recipient has or have altruistic utility, the transfer paradox can never take place irrespective of whether utility is defined to include altruism. Altruism does not cause the donor’s enrichment. Therefore, our result assures that the benevolent assumption that the donor country has altruistic intentions toward the recipient country cannot elucidate the reason why the donor gives aid voluntarily. As the existing literature has theorized, the distortion that hinders free trade is necessary for the transfer paradox to take place in the model with two countries and two goods.

The remainder of this paper is organized as follows. Section 2 describes the model wherein the donor and/or recipient has or have altruistic utility in the model with two countries and two goods. Sections 3 and 4 present the main results. In Section 3 (Section 4), we analyze the transfer paradox with regard to the utility that includes (resp. excludes) altruism. Section 5 concludes the paper with some remarks.

2 The model

Consider a general equilibrium model of international trade in a two-commodity world involving two countries. There are two countries — a donor country (indexed by \( \alpha \)) and a recipient country (indexed by \( \beta \)). They trade in two goods — the non-numeraire good \((x_1)\) and the numeraire good \((x_2)\). Suppose that the donor (recipient) is an exporter (resp. importer) of the non-numeraire good. It is assumed that foreign aid is distributed in lump-sum among consumers. There is no import tariff or export subsidy. \( T \geq 0 \) denotes the transfer as foreign aid. The donor provides foreign aid of the amount \( T \) in terms of the numeraire good to the recipient. \( p \) represents the international price of the non-numeraire good, which could be interpreted as a relative price because unity constitutes the domestic price of the numeraire good.

The consumption pair of the representative consumer in country \( i = \alpha, \beta \) (hereafter consumer \( i \)) is denoted by \((x^i_1, x^i_2)\), where \( x^i_1 \) is the non-numeraire good and \( x^i_2 \) is the numeraire good. Define the utility that consumer \( i \) obtains directly from the consumption of goods by \( u^i \equiv \)
$u^i(x_1^i, x_2^i)$. We denote $u^i$ as “self-utility.” Self-utility does not include any altruistic part of utility. Consumer $\alpha$ ($\beta$) obtains the utility $u^\alpha = u^\alpha(x_1^\alpha, x_2^\alpha)$ (resp. $u^\beta = u^\beta(x_1^\beta, x_2^\beta)$) by consuming goods. In order to describe altruism for the people in both countries, we define the “total utility” of consumer $i$ by $U^i = U^i(u^i, u^j) = U^i(u^i(x_1^i, x_2^i), u^j(x_1^j, x_2^j)), i, j = \alpha, \beta, j \neq i$. It should be noted that the total utility of consumer $i$ includes the altruistic utility that is raised by the increase in $u^j$. For simplification of analysis, we assume that self-utility and total utility are continuously differentiable. Under differentiability, altruism implies that $\frac{\partial U^i}{\partial u^j} \geq 0$ is satisfied. Moreover, we assume that the impact on the total utility of the self-utility always exceeds that of the altruistic part. In other words, we assume the following.

**Assumption 1:** $\frac{\partial U^i}{\partial u^i} > \frac{\partial U^i}{\partial u^j} \geq 0$.

Denote trade expenditure function by $E^i$, which is defined as the difference between the expenditure function $e^i$ and revenue function $r^i$. Thus, the following equations are satisfied:

$$E^\alpha(p, U^\alpha) \equiv e^\alpha(p, U^\alpha) - r^\alpha(p), \quad (1)$$
$$E^\beta(p, U^\beta) \equiv e^\beta(p, U^\beta) - r^\beta(p). \quad (2)$$

We denote the import demand function of the non-numeraire good in country $i$ by $m^i$. Without a loss of generality, we assume that $m^\alpha < 0$ and $m^\beta > 0$. Thus, the recipient imports the non-numeraire good by $m^\beta(> 0)$.

The budget constraints in the countries are as follows:

$$E^\alpha(p, U^\alpha) = -T, \quad (3)$$
$$E^\beta(p, U^\beta) = T. \quad (4)$$

The product market-clearing condition is as follows:\(^3\)

$$m^\alpha(p, U^\alpha) + m^\beta(p, U^\beta) = 0. \quad (5)$$

\(^3\)The world market-clearing condition for the numeraire good has been omitted due to Walras’s law.
Using McKenzie’s lemma, the following equation is satisfied:\(^4\)

\[ m^i = E^i_p. \]  

(6)

3 Total utility and the transfer paradox

In this section, we examine whether the transfer paradox takes place with regard to the total utility by applying the argument of Walrasian stability. Let us investigate the impact of an increase in the unfettered transfer \( T \) upon the variables of the model described above \((U^\alpha, U^\beta, p)\).

The total differentiation of eqs. (3)–(5) provides the following equation:

\[
\begin{bmatrix}
E^\alpha_U & 0 & m^\alpha \\
0 & E^\beta_U & m^\beta \\
m^\alpha_U & m^\beta_U & M_p
\end{bmatrix}
\begin{bmatrix}
dU^\alpha \\
dU^\beta \\
dp
\end{bmatrix}
= \begin{bmatrix}
-1 \\
1 \\
0
\end{bmatrix} dT,
\]

(7)

where \( M_p \equiv m^\alpha_p + m^\beta_p < 0 \). Applying Cramer’s rule to the equation of total differentiation (7), we obtain the following equations.

\[
\frac{dU^\alpha}{dT} = \frac{M_p - (m^\alpha + m^\beta)m^\beta_U(U^\alpha_U)^{-1}}{\Delta E^\alpha_U}, \quad (8)
\]

\[
\frac{dU^\beta}{dT} = \frac{-M_p + (m^\alpha + m^\beta)m^\alpha_U(U^\beta_U)^{-1}}{\Delta E^\beta_U}, \quad (9)
\]

\[
\frac{dp}{dT} = \frac{m^\beta_U(U^\beta_U)^{-1} - m^\alpha_U(U^\alpha_U)^{-1}}{\Delta}, \quad (10)
\]

where \( \Delta \equiv -M_p + m^\alpha m^\alpha_U(U^\alpha_U)^{-1} + m^\beta m^\beta_U(U^\beta_U)^{-1} \). Substituting \( m^\alpha + m^\beta = 0 \) into (8) and (9), we immediately obtain the following equations.

\[
\frac{dU^\alpha}{dT} = \frac{M_p}{\Delta E^\alpha_U}, \quad (11)
\]

\[
\frac{dU^\beta}{dT} = \frac{-M_p}{\Delta E^\beta_U}, \quad (12)
\]

Samuelson (1952) shows that if the general equilibrium satisfies the Walrasian stability condition, it is not possible for any transfer paradox to occur in the model with two countries.

\(^4\) The subscript \( x \) represents the partial derivative of the functions with respect to \( x \).
and two goods. Applying a similar argument to the total utility \( U^i \), we obtain the following proposition.

**Proposition 1.** If the Walrasian stability condition is satisfied, it is not possible for any transfer paradox with regard to the total utility to occur in the model with two countries and two goods. That is, \( \frac{dU^\alpha}{dT} < 0 \) and \( \frac{dU^\beta}{dT} > 0 \) are satisfied.

**Proof.** The proof follows directly from Brakman and Marrewijk (1999, Ch.3). Denote \( \dot{p} \) as the change in the price of the non-numeraire good \( x_1 \) over time as a result of an imbalance in the demand and supply of good \( x_1 \). As the Walrasian stability condition, consider (3), (4), and the following dynamic adjustment equation:

\[
\dot{p} = \Pi (m^\alpha (p, U^\alpha) + m^\beta (p, U^\beta)).
\]

Since the function \( \Pi(\cdot) \) in (13), which is assumed to be continuously differentiable, depends on the world excess demand of good \( x_1 \), we assume that the price of good \( x_1 \) is rising if and only if the world excess demand for good \( x_1 \) is positive, such that \( \Pi(0) = 0 \) and \( \Pi'(0) > 0 \). If we linearize systems (3), (4), and (13) around equilibrium values of price and utility, say \((\bar{p}, U^\alpha, U^\beta)\), and use the normalization above, we obtain:

\[
m^\alpha (p - \bar{p}) + E^\alpha_U (U^\alpha - \bar{U}^\alpha) = 0, \tag{14}
\]

\[
m^\beta (p - \bar{p}) + E^\beta_U (U^\beta - \bar{U}^\beta) = 0, \tag{15}
\]

\[
\dot{p} = \Pi(0) + \Pi'(0) \left[ m^\alpha_p (p - \bar{p}) + m^\alpha_U (U^\alpha - \bar{U}^\alpha) + m^\beta_p (p - \bar{p}) + m^\beta_U (U^\beta - \bar{U}^\beta) \right]. \tag{16}
\]

By (14) and (15), \( U^\alpha - \bar{U}^\alpha = -\frac{m^\alpha}{E^\alpha_U} (p - \bar{p}) \) and \( U^\beta - \bar{U}^\beta = -\frac{m^\beta}{E^\beta_U} (p - \bar{p}) \) are obtained. Substituting them into (16) and using \( \Pi(0) = 0 \) gives

\[
\dot{p} = \Pi'(0) \left[ m^\alpha_p - \frac{m^\alpha m^\alpha_U}{E^\alpha_U} + (m^\beta_p - \frac{m^\beta m^\beta_U}{E^\beta_U}) \right] (p - \bar{p}) = -\Pi'(0) \Delta (p - \bar{p}). \tag{17}
\]

For (local) Walrasian stability, we want the price change of good \( x_1 \) to be negative if \( p \) exceeds the equilibrium price \( \bar{p} \) and to be positive if \( p \) falls short of the equilibrium price \( \bar{p} \). Walrasian stability thus requires that \( \Delta > 0 \). As \( M_p < 0 \) and \( E^\alpha_U e^i_U > 0 \) are satisfied, \( \frac{dU^\alpha}{dT} = \frac{M_p}{\Delta E^\alpha_U} < 0 \) and \( \frac{dU^\beta}{dT} = -\frac{M_p}{\Delta E^\beta_U} > 0. \)
Proposition 1 implies that even if the altruistic utility is introduced into the model, no transfer paradox occurs in the world in which Walrasian stability is guaranteed. In other words, as regards the total utility level \( U^i \equiv U^i(u^i, u^j) \), the donor can never enrich and the recipient can never immiserize by the transfer from the donor to the recipient. From this proposition, the motivation for the donor to give aid cannot be explained by its enrichment as a result of altruism. Moreover, it should be noted that Proposition 1 does not depend on Assumption 1. Thus, the above result remains to be seen in other general cases in which both utilities have various externalities. For example, even when the utility is not altruistic but negatively influenced — for example, when the utility is enviable — Walrasian stability guarantees that there is no transfer paradox with regard to the total utility.

The existing literature concerning altruism has not analyzed the relationship between total utility and the transfer paradox in an explicit manner and has been concerned about the impact that altruism has on self-utility excluding the altruistic part. In the next section, we investigate whether transfer paradox takes place with regard to self-utility.

4 Self-utility and the transfer paradox

As it is assumed that self-utility and total utility are continuously differentiable, the following equations are satisfied by Proposition 1.

\[
\frac{dU^\alpha}{dT} = \frac{\partial U^\alpha}{\partial u^\alpha} \frac{\partial u^\alpha}{\partial T} + \frac{\partial U^\alpha}{\partial u^\beta} \frac{\partial u^\beta}{\partial T} < 0, 
\]

\[
\frac{dU^\beta}{dT} = \frac{\partial U^\beta}{\partial u^\alpha} \frac{\partial u^\alpha}{\partial T} + \frac{\partial U^\beta}{\partial u^\beta} \frac{\partial u^\beta}{\partial T} > 0, 
\]

The signs of \((\frac{\partial u^\alpha}{\partial T}, \frac{\partial u^\beta}{\partial T})\) are classified into the following four cases: \((+, +), (+, -), (-, +), \) and \((-,-)\). However, only the case wherein \((\frac{\partial u^\alpha}{\partial T}, \frac{\partial u^\beta}{\partial T}) = (-, +)\) is supported under Assumption 1. We summarize the result in the following proposition.

**Proposition 2.** Suppose that Walrasian stability and Assumption 1 are satisfied. As regarding self-utility, even if the donor and/or the recipient has or have altruistic utility, no transfer
paradox occurs in the model with two countries and two goods. That is, \( \frac{\partial u^\alpha}{\partial T} < 0 \) and \( \frac{\partial u^\beta}{\partial T} > 0 \) are satisfied.

**Proof.** As \( (\frac{\partial U^\alpha}{\partial u^\alpha}, \frac{\partial U^\beta}{\partial u^\beta}) \geq (0, 0) \) under Assumption 1, in order for both inequalities (18) and (19) to be satisfied, the cases wherein \( (\frac{\partial u^\alpha}{\partial T}, \frac{\partial u^\beta}{\partial T}) = (+, +) \) and \( (-, -) \) are excluded. Thus, possible cases are limited to the cases wherein \( (\frac{\partial u^\alpha}{\partial T}, \frac{\partial u^\beta}{\partial T}) = (+, -) \) and \( (-, +) \). Consider the case wherein \( \frac{\partial U^\beta}{\partial u^\alpha} = 0 \). As \( \frac{\partial u^\beta}{\partial T} > 0 \) is satisfied by (19), in order for (18) to be satisfied, \( \frac{\partial u^\alpha}{\partial T} < 0 \) is required to be satisfied. Likewise, in the case wherein \( \frac{\partial U^\alpha}{\partial u^\beta} = 0 \), \( \frac{\partial u^\alpha}{\partial T} < 0 \) must be satisfied by (18). By (19), \( \frac{\partial u^\beta}{\partial T} > 0 \) is required to be satisfied. Consider the case wherein \( (\frac{\partial U^\alpha}{\partial u^\alpha}, \frac{\partial U^\beta}{\partial u^\alpha}) > (0, 0) \). Multiplying (18) and (19) by \( \frac{\partial U^\alpha}{\partial u^\alpha} \) and \( \frac{\partial U^\beta}{\partial u^\beta} \), respectively, and adding both inequalities, we obtain \( \left( \frac{\partial U^\alpha}{\partial u^\alpha} \frac{\partial U^\beta}{\partial u^\beta} - \frac{\partial U^\alpha}{\partial u^\beta} \frac{\partial U^\beta}{\partial u^\alpha} \right) \frac{\partial u^\alpha}{\partial T} > 0 \). By Assumption 1, \( \frac{\partial u^\alpha}{\partial T} < 0 \) is satisfied. Likewise, multiplying (18) and (19) by \( \frac{\partial U^\alpha}{\partial u^\beta} \) and \( \frac{\partial U^\alpha}{\partial u^\alpha} \), respectively, and adding both inequalities, we obtain \( \left( \frac{\partial U^\alpha}{\partial u^\alpha} \frac{\partial U^\beta}{\partial u^\beta} - \frac{\partial U^\alpha}{\partial u^\beta} \frac{\partial U^\beta}{\partial u^\alpha} \right) \frac{\partial u^\beta}{\partial T} > 0 \). By Assumption 1, \( \frac{\partial u^\beta}{\partial T} > 0 \) is satisfied. \( \square \)

Proposition 2 implies that no transfer paradox takes place with regard not only to the total utility, but also self-utility. Even if there is altruism for the donor and/or recipient, the donor does not raise its self-utility by giving aid and the recipient’s self-utility does not fall. The assertion of Proposition 2 seems surprising at first glance, because even if people give aid to others, they cannot raise their own welfare through this help-providing action. Even if the donor is concerned about the rise in the utility of the recipient, the former cannot become happier by supporting the recipient through the transfer. Proposition 2 concludes that in order for the transfer paradox to take place, other distortions — which have already been analyzed in the existing literature — are required to be introduced into the model. Therefore, although Lahiri and Raimondos-Møller (1999) Takarada and Tawada (2003) explore the model in which altruism is introduced along with other distortions such as trade barriers or political objectives, they cannot explain transfer paradox if there is no distortion in the model.

Unlike Proposition 1, the proof of Proposition 2 depends on Assumption 1. Thus, if Assumption 1 is not assumed, there exists a possibility of the transfer paradox occurring. However, the
violation of Assumption 1 is quite unrealistic in the sense that it is based on the assumption that the people in a country emphasize the welfare of the other country more than their own. Therefore, for example, although Kemp and Shimomura (2002) show the possibility of a Pareto-improving transfer, in order for both the countries to benefit from the transfer, it is necessary that the irregular assumption about the impact of altruistic utility is satisfied. If a condition similar to Assumption 1 is assumed, the possibility of a Pareto-improving transfer will be excluded. As long as the consumers of both the donor and recipient countries are predominantly concerned about the maximization of their own utility, the transfer necessarily causes a decrease in the donor’s welfare and an increase in the recipient’s welfare.

5 Concluding remarks

In the paper, we challenged the conventional wisdom that suggests that altruism motivates the donor country to give aid to the recipient. We demonstrated that in the model with two countries and two goods, even if the donor and/or recipient has or have altruistic utility, the transfer paradox can never take place with regard to both the total utility and self-utility. Although the existing literature that focuses only on selfish utility has emphasized that if Walrasian stability is guaranteed, no transfer paradox can take place, a similar result — of the impossibility of the transfer paradox — is achieved even if the extended utility is allowed to include altruism. As a result, the reason behind the donor country’s transfer of economic aid to the recipient cannot be explained by the externality between the utilities of the donor and recipient, such as altruism. Irrespective of the existence of altruism, the donor country must sacrifice its own welfare for the improvement of the welfare of the other country.

The result of the paper implies that the contention that altruism can raise the welfare of the people in the donor country is just an illusion. The paper suggests that the motivation of compassionate or charity cannot justify the transfer activities by the people in developed countries as economic aid from donor countries to developing countries. Even if the people in
developed countries possess a merciful disposition toward the people in poor countries — apart from having the economic abundance necessary for providing economic aid to these people — they cannot drive themselves to help toward the cause of poverty reduction in developing countries.
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