Alternative Strategies of a Public Enterprise in Oligopoly Revisited:
An Extension of Stackelberg Competition

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Series No.168
This paper revisits De Fraja and Delbono (1989), which is the seminal paper on mixed oligopoly, in order to pay more attention to Stackelberg competition. First, we show that, even in Cournot competition, if the number of private firms is sufficiently small, privatization necessarily reduces social welfare. Second, we demonstrate that when a public firm is a Stackelberg leader before and after privatization, privatization necessarily reduces welfare irrespective of the number of private firms. Moreover, we show that even when a public firm remains a follower, privatization reduces welfare if the number of private firms is relatively small.

**Keywords:** Mixed oligopoly; privatization; Stackelberg leader and follower; welfare analysis

**JEL classification numbers:** D43, L13, L33

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We wish to thank Mitsuyoshi Yanagihara for their valuable suggestions. This study was supported in part by MEXT/JSPS Grant-in-Aid for Scientific Research (C) No. 25380286 (Kojun Hamada). The usual disclaimer applies.
I INTRODUCTION

Privatization has been a global trend in capitalist market economies. However, in recent years, mixed oligopolies have been common, in which a public firm and private firms engage in competition in the same market. Does privatization improve welfare? To answer this question, the seminal paper of De Fraja and Delbono (1989) compared welfare before and after privatization in a mixed oligopoly. They demonstrated that whether privatization improves welfare depends on the number of private firms; in particular, when the number of private firms is relatively small (large), privatization deteriorates (improves) welfare.

This paper revisits the work of De Fraja and Delbono (1989), who first examined the oligopolistic competition between a public firm and private firms before and after privatization, in order to examine the Stackelberg competition between them. Although De Fraja and Delbono (1989) provided the remarkable result that privatization can improve welfare even if a public firm changes its maximized objective from welfare to profit, they limited their analysis mainly to Cournot competition between a public firm and private firms. In addition, although they also considered Stackelberg competition in which a public firm is a leader before privatization, they only compared this Stackelberg competition before privatization and Cournot competition after privatization.

This paper extends the analysis of De Fraja and Delbono (1989) and others by allowing Stackelberg competition before and after privatization. In particular, unlike the existing literature in which a comparison is made between Stackelberg competition when a public firm is a leader before privatization and Cournot competition after privatization, we compare social welfare before and after privatization when a public firm remains either a Stackelberg leader or follower.

Many studies have examined mixed oligopolies. However, relatively few articles have examined mixed oligopolies in a Stackelberg competition. One exception is Beato and Mas-Colell (1984), who examined a mixed oligopoly in which a private and a public firm compete in a market including both Cournot and Stackelberg competition. They demonstrated the result that when the timing of output choice is altered, the relative size of social welfare is also changed. However, they only focused on duopolistic competition between a public and a private firm and do not compare the welfare change resulting from privatization. Pal (1998) and Fjell and Heywood (2004) analysed the endogenous timing of moves in a mixed oligopoly, but did not conduct a welfare comparison. Furthermore, other studies have paid little attention to the link between the timing of decision making and welfare before and after privatization (e.g., Matsumura, 2003a, 2003b; Jacques, 2004; Lu, 2006, 2007).

In this paper, we first show that even in Cournot competition, if the number of private firms is suf-

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1 De Fraja and Delbono (1990) reviewed various mixed oligopoly models, including different move orders in oligopolistic games. For a recent survey on mixed oligopoly, see Matsumura and Shimizu (2010).
iciently small, privatization necessarily reduces social welfare. Then we compare the welfare before and after privatization when a public firm remains a Stackelberg leader or follower. We demonstrate that when a public firm remains a Stackelberg leader before and after privatization, privatization necessarily reduces welfare irrespective of the number of private firms. This result is in stark contrast with that of De Fraja and Delbono (1989) who claimed that privatization might bring about welfare improvement. Moreover, we show that even when a public firm is a Stackelberg follower, privatization decreases welfare if the number of private firms is relatively small. The results suggest that the situation in which privatization in a mixed oligopoly enhances welfare is quite limited and this presents a rationale as to why mixed oligopolies exist in many countries.

The remainder of this paper is organized as follows. Section II describes the mixed oligopoly model wherein a public firm and private firms compete in a market. Section III presents the results of a welfare comparison before and after privatization in Cournot and Stackelberg competition. Section IV concludes the paper with some final remarks.

II THE MODEL

The basic setting follows that of De Fraja and Delbono (1989). There are \((n+1)\) firms that have an identical production technology in a simple homogeneous product market. One firm is a public firm and the others are private firms. They engage in quantity competition in the oligopolistic market. The objective of the public firm is to maximize social welfare and the objective of \(n\) private firms is to maximize their own firm’s profit. The public firm is indexed by firm \(i = 0\) and private firms are indexed by firm \(i = \{1, 2, \ldots, n\}\). \(q_i\) denotes firm \(i\)’s output. As we focus only on the symmetric equilibrium among private firms, the output of identical private firms is the same, i.e., \(q \equiv q_i, i = \{1, \ldots, n\}\). The inverse demand function is given by \(p = p(Q) = a - Q, a > 0\), where \(p\) denotes the price and \(Q = q_0 + nq\) the total output. The cost function of firms is denoted by \(C(q_i) = c + \frac{k}{2}q_i^2, c \geq 0, k > 0\). For brevity, we assume that \(c = 0\). The profit of firm \(i\) is denoted by \(\pi_i = p(Q)q_i - \frac{k}{2}q_i^2\). We denote the profit of identical private firms as \(\pi \equiv \pi_i, i = \{1, \ldots, n\}\). Consumer surplus and producer surplus are denoted by \(CS \equiv \int_0^Q p(s)ds - p(Q)Q = \frac{1}{2}Q^2\) and \(PS \equiv \pi_0 + n\pi = p(Q)Q - \frac{k}{2}\sum_{i=0}^n q_i^2\), respectively. Social welfare is defined as \(W \equiv CS + PS = aQ - \frac{1}{2}Q^2 - \frac{k}{2}\sum_{i=0}^n q_i^2\). For brevity, we ignore the integer problem on the number of firms.

We classify the timing of output choice by a public firm and private firms as follows. First, in order to review the result of De Fraja and Delbono (1989), we consider Cournot competition in which all firms choose their own output in a simultaneous-move game. Then we proceed to investigate Stackelberg competition in a sequential-move game in which a public firm is the Stackelberg leader or follower.
III RESULTS

First, in Subsection III.1, we summarize the welfare comparison result of De Fraja and Delbono (1989) before and after privatization in Cournot competition. Second, in Subsection III.2, we compare welfare in a Stackelberg competition in which the public firm remains a leader before and after privatization. Third, in Subsection III.3, we compare welfare in a Stackelberg competition in which the public firm remains a follower before and after privatization.

III.1 Cournot competition

One of the most important contributions of De Fraja and Delbono (1989) was a welfare comparison before and after privatization when a public firm and private firms engage in Cournot competition. We summarize their results about the variables in equilibrium in TABLE 1.

[Insert TABLE 1 here]

As shown in TABLE 1, De Fraja and Delbono (1989) compared welfare before and after privatization in Cournot competition between a public firm and private firms.\(^2\) When we denote the welfare before and after privatization by \(W^I\) and \(W^II\), respectively, their welfare comparison result can be summarized in the following proposition.

**Proposition 1.** (De Fraja and Delbono (1989))

Consider the situation in which the public firm and private firms engage in Cournot competition before and after privatization. Welfare before privatization is larger (smaller) than that after privatization when the number of private firms is relatively small (large). Strictly stated, \(W^I \gtrless W^II\) if and only if \(n \leq n^*(k) = \frac{-k + \sqrt{k^2 + 4k(1+k)^3}}{2k} (> 0)\).

**Proof.** The proof follows by direct calculation. \(\square\)

Proposition 1 insists that there exists \(n^*(k) \in \mathbb{R}_+\), such that \(W^I > W^II\) if \(n < n^*(k)\) and vice versa. However, De Fraja and Delbono (1989) did not investigate how large this threshold \(n^*(k)\) will be. Therefore, when we scrutinize the size of this threshold, we obtain the following proposition regarding the welfare comparison as a corollary of Proposition 1.

**Proposition 2.** If the number of private firms is one or two, welfare before privatization is necessarily larger than that after privatization. That is, if \(n = 1\) or \(2\), \(W^I > W^II\) for all \(k\).

\(^2\) Note that a typographical error exists in the public firm’s profit \(z_0\) before privatization in Table 1 (p.306) of De Fraja and Delbono (1989). The correct value is not \(\frac{a^2(k+1)^2}{2z^2}\), but \(\frac{a^2(k+1)^2}{2z^2}\), as shown in TABLE 1.
Proof. Consider \(\min_k n^*(k)\), which denotes the minimum value of the threshold \(n^*(k) = \frac{-k+\sqrt{k^2+4k(1+k)^3}}{2k}\) \((>0)\). It is calculated as \(n^*(k^*) = \sqrt{7} - \frac{1}{2} \approx 2.1458\) when \(k^* = \frac{1}{2}\). Thus, it necessarily holds that when \(n = 1, 2, n < n^*(k) \forall k\). As \(n < n^*(k) \forall k\) when \(n = 1, 2, W^I > W^{II}\) immediately follows. \(\Box\)

Proposition 2 implies that when a public firm and a private firm (or two private firms) engage in Cournot competition, the public firm should not be privatized, irrespective of the coefficient of marginal cost, \(k > 0\). This finding does not appear in De Fraja and Delbono (1989) in an explicit manner. The threshold \(n^*(k)\) as a function of \(k\) is graphed in FIGURE 1, which shows that the lower bound of the threshold is 2.1458.

In Figure 2 (p.309) of De Fraja and Delbono (1989), both welfare before and welfare after privatization, which we denote as \(W^I\) and \(W^{II}\), respectively, are depicted as single-peaked functions of the number of private firms \(n\). However, it should be noted that the single-peaked configuration of the functions is not correct. The derivative of \(W^I\) with respect to \(n\) is calculated as \(\frac{dW^I}{dn} = \frac{k^2(k+2)(k+1)^2+k^2n}{(1+k)^3+nk^2} a^2 > 0\) and the limits of welfare and its derivative are \(\lim_{n \to \infty} W^I = \frac{a^2}{2}\) and \(\lim_{n \to \infty} \frac{dW^I}{dn} = 0\), respectively. Thus, \(W^I\) is a monotonically increasing function of \(n\) and as \(n\) approaches infinity, the value converges to unity (the slope converges to zero). Likewise, the derivative of \(W^{II}\) is calculated as \(\frac{dW^{II}}{dn} = \frac{k^2+4k^2+2kn}{(2+k+n)^3} a^2 > 0\) and the limits of welfare and its derivative are \(\lim_{n \to \infty} W^{II} = \frac{a^2}{2}\) and \(\lim_{n \to \infty} \frac{dW^{II}}{dn}\), respectively. Thus, \(W^{II}\) is also a monotonically increasing function of \(n\) and as \(n\) approaches infinity, the value converges to \(\frac{a^2}{2}\) (the slope converges to zero). The configurations of \(W^I\) and \(W^{II}\) with respect to \(n\) are depicted in FIGURE 2.

III.2 Stackelberg competition when a public firm is the leader

De Fraja and Delbono (1989) compared welfare between Cournot competition and Stackelberg competition. To be more precise, they compared the welfare in Stackelberg competition when a public firm is a leader before privatization with that in Cournot competition after privatization. They assumed that after privatization, the public firm loses the Stackelberg leader’s position regarding the timing of output choice and they obtained the result that the welfare before privatization is always greater than that after privatization.

However, another comparison can be made in a situation not considered by De Fraja and Delbono (1989). In particular, they did not investigate the situation in which the public firm is a Stackelberg leader both before and after privatization. By assuming that the public firm loses its leader position after privatization, privatization has two different effects for the public firm. One is the change in the
public firm’s objective and the other is the loss of leadership position. They showed that both effects bring about lower welfare after privatization. Therefore, it is necessary to distinguish between these two different effects brought about by privatization. In the following, we consider the situation in which the public firm changes only the objective to be maximized, that is, changing from welfare to its own profit, before and after privatization, while there is no change in the Stackelberg leader’s position for the public firm. Unlike the existing literature, we can examine only the effect of the change in the public firm’s objective associated with privatization.

In the situation in which a public leader firm and private follower firms engage in Stackelberg competition before and after privatization, we summarize the calculation result about the variables in equilibrium in TABLE 2.

Note that De Fraja and Delbono (1989) compared welfare between $W^{III}$ and $W^{II}$. In other words, they compared welfare in pre-privatization Stackelberg competition with that in post-privatization Cournot competition. Their comparison result is summarized in the following proposition.

**Proposition 3.** (De Fraja and Delbono (1989))

Consider the situation in which the public firm acts as a Stackelberg leader before privatization, while all firms engage in Cournot competition after privatization. The welfare before privatization is necessarily larger than that after privatization, irrespective of the number of private firms. That is, $W^{III} > W^{II}$ for all $n$.

**Proof.** By simple calculation, the following equation is satisfied:

$$W^{III} > W^{II} \iff \frac{t + kn(1 + \beta)}{(t + k\beta^2)} > \frac{(3 + k)(t + k\beta^2) + (4 + k)n(t + k\beta^2) + n^2(t + k\beta^2)}{(1 + \beta)^2} \iff (1 + \beta)^2 > 0.$$ 

Proposition 3 implies that the welfare before privatization is necessarily higher than that after privatization.³ In this comparison, however, privatization in this situation includes two different effects: the changes in the public firm’s objective and the timing of output choice. To be precise, their result showed that the welfare before privatization, when the public firm is the Stackelberg leader and maximizes the social welfare, is necessarily higher than that after privatization when the public firm competing with private firms in Cournot competition maximizes its own profit. Therefore, the effect in which the public firm loses the position of the Stackelberg leader after privatization might cause this result. Now, we pay attention only to the change in the public firm’s objective from welfare maximization to profit maximization before and after privatization without changing any timing of output choice, and make a welfare

³ As already mentioned regarding $W^I$ and $W^{II}$, although De Fraja and Delbono (1989) also depicted $W^{III}$ as a single-peaked function of $n$, $W^{III}$ is a monotonically increasing function of $n$. The derivative of $W^{III}$ is $\frac{dW^{III}}{dn} = \frac{(k+2)(k+1)^2 + n(kn+2(k+1)^2)}{(t + k\beta^2)^2} \epsilon_{III} > 0$ and the limits of welfare and its derivative are $\lim_{n \to \infty} W^{III} = \frac{a^2}{2}$ and $\lim_{n \to \infty} \frac{dW^{III}}{dn} = 0$, respectively. As $n$ approaches infinity, $W^{III}$ converges to $a^2/2$ (the slope converges to zero). See also FIGURE 1.
comparison in Stackelberg competition before and after privatization. We denote the welfare before and after privatization in this Stackelberg case, as shown in TABLE 2, by $W^{III}$ and $W^{IV}$, respectively.\footnote{By tedious calculation, $W^{IV}$ is rearranged as $W^{IV} = \frac{A k^2}{2p(k+4)^2}$, where $A \equiv k^2 n^4 + k(k+2)(3k+2)n^3 + (k+1)^2(3k^2+11k+4)n^2(k+1)^3(k^2+7k+8)n + (k+1)^4(k+3)$.} We provide the welfare comparison result in the following proposition.

**Proposition 4.** Consider the situation in which the public firm acts as a Stackelberg leader before and after privatization. The welfare before privatization is not less than that after privatization, irrespective of the number of private firms. That is, $W^{III} \geq W^{IV} \forall n$.

**Proof.** By simple calculation, the following equation is satisfied: $W^{III} > W^{IV}$

$$\frac{t+kn(1+\beta)}{n^2} > \frac{(1+\beta)(1+k)n}{(1+\beta)(1+k)^2} \Leftrightarrow \beta^2(1+k+t)^2((1+\beta)^2(k^2+1)k^3) > 0 \Rightarrow ((k+1)^3 - kn^2)^2 > 0. \text{ The equality holds only when } n = \sqrt{\frac{(k+1)^3}{k}}.$$

Proposition 3 mentions that if the public firm acts as the Stackelberg leader even after privatization, the welfare before privatization is not less than that after privatization. Even though the public firm maintains the leader position when choosing the output, the change in the firm’s maximized objective from welfare to profit reduces welfare. It suggests that the possibility that welfare increases after privatization is limited to quite a restrictive case in which the simultaneous-move game à la Cournot competition is brought about only when the number of private firms is relatively large.

At first glance, it seems that welfare decreases after privatization when the public firm loses the Stackelberg leader position. However, this is not necessarily true. Compare the welfare after privatization when the public firm competes in a Cournot manner and when it acts as the Stackelberg leader, that is, $W^{II}$ and $W^{IV}$. As it turns out, which welfare after privatization is larger depends on the relative size of $(n,k)$. The following relationship is satisfied: $W^{II} \geq W^{IV}$ if and only if

$$\frac{(3+k)(k+4)n+n^2}{(1+\beta)^2} > \frac{(1+\beta)(1+k)n}{(1+\beta)(1+k)^2}.$$  

Moreover, the last inequality holds if and only if $B(n,k) \equiv n(kn^2(n+1) - 3(k+1)^3n - 2(k+2)(k+1)^3) \geq 0$. By direct calculation, $\frac{\partial B(n,k)}{\partial n} = kn(3n^2 + 3(k+1)^3)$ and $\frac{\partial B(n,k)}{\partial n}|_{n=1} < 0$ hold. As $n$ becomes sufficiently large with $k$ being constant, the sign of $\frac{\partial B(n,k)}{\partial n}$ turns from negative to positive. In other words, there exists $n^*$, such that $\frac{\partial B(n,k)}{\partial n} = 0$, which is defined by $n^* = \frac{-k + \sqrt{k^2 + 9k(k+1)}}{3k}$ ($> 0$). Therefore, when the number of private firms $n$ is sufficiently large under a given $k$, i.e., when $n > n^*$, $W^{II} > W^{IV}$ is likely to occur.

The welfare comparison between $W^{II}$ and $W^{IV}$ is equivalent to the welfare comparison in an oligopolistic competition among $(n+1)$ firms between when the simultaneous-move Cournot competition occurs and when a firm is a Stackelberg leader. Stated differently, the comparison is made only in terms of the difference in the timing of output choice. Which instance of welfare is greater depends on the number
of private firms and the marginal cost. As the number of private firms increases, the welfare in Cournot competition tends to be larger than that in Stackelberg competition. The reason is that as the number of firms is relatively large, Cournot competition in the simultaneous-move game becomes more competitive than Stackelberg competition in the sequential-move game and harsh competition brings about greater welfare, as suggested by the Cournot limit theorem. In contrast to Cournot competition, as the leader continues to retain some degree of monopoly power in Stackelberg competition even if the number of follower firms increases, less intense competition leads to lower welfare.

### III.3 Stackelberg competition when a public firm is the follower

In this subsection, we consider the opposite case to the one analysed in Subsection III.2; here, the public firm is the Stackelberg follower before and after privatization. As already investigated in Subsection III.2, it is often assumed that the public firm has an advantage over the private firms with regard to the timing of output choice. So far, however, no economically persuasive rationale as to why the public firm must act as a Stackelberg leader has been presented in an explicit manner in the existing literature. On the contrary, the possibility that the public firm acts as a Stackelberg follower might exist because it has often been pointed out that the public firm has slower decision-making capabilities than the private firm. Furthermore, surprisingly, it is possible that the welfare before privatization when the public firm acts as a Stackelberg follower is greater than that when it competes in a Cournot fashion, or more interestingly, even when it acts as a Stackelberg leader (we will show this result in the following analysis).

Therefore, we consider the situation in which a public follower firm and private leader firms engage in Stackelberg competition before (and also after) privatization. We summarize the calculation result about the variables in equilibrium in TABLE 3.

![Insert TABLE 3 here]

We denote the welfare before and after privatization in Stackelberg competition when the public firm acts as a follower, as shown in TABLE 3, by \( W^V \) and \( W^{VI} \), respectively.

Before making a welfare comparison before and after privatization in this case, we provide the result of the welfare comparison before privatization of all three cases with regard to the timing of output choice. Comparing the three kinds of welfare in Cournot competition and Stackelberg competition when the public firm is a leader or a follower, i.e., \( W^I \), \( W^{III} \), and \( W^V \), we obtain the following proposition.

**Proposition 5.** Compare the welfare before privatization when the public firm engages in Cournot competition and when it acts as a Stackelberg leader or a follower. The welfare before privatization when the
public firm is a follower is larger than when it is a leader, and it is larger than when it faces Cournot competition. These results are satisfied irrespective of the number of private firms. That is, $W^V > W^{III} > W^I$ for all $n$.

**Proof.** By direct calculation, the following equations are satisfied: $W^V > W^{III} \iff \frac{(1+k)n^2+(1+k)(4+k)n+(2+k)^2}{(1+k)(1+k)^2} > t^2 \iff kn(n+2k+3) > 0$ and $W^{III} > W^I \iff \frac{t+kn(1+\beta)}{t+(k+\beta)^2} > (1+k)^3+nk(2+4k+k^2) + (2+k)^2 > (1+k)((3+k) + (4+k)n + n^2) \iff (2+k)^2 - (1+k)(3+k) = 1 > 0. \square$

As is usually expected, the public firm as the Stackelberg leader acquires the first-mover advantage by moving from the simultaneous choice of output to the first-mover choice. In this sense, it is appropriate that the existing literature assumes the public firm is the first mover. However, in the mixed oligopolistic competition, unlike the pure competition between private firms, the public firm as the Stackelberg follower can acquire larger welfare than when it is the leader. The result that the public firm can enjoy the second-mover advantage is quite counterintuitive. The reason why this result arises is that the second-mover welfare maximizer can adjust total quantity more appropriately by observing the outputs of the first-mover private firms. Therefore, the result of Proposition 5 justifies our setting in which the public firm becomes a Stackelberg follower.

First, like the comparison of De Fraja and Delbono (1989) of Cournot and Stackelberg competition, we make a welfare comparison between $W^V$ and $W^{II}$. In other words, we compare the welfare before privatization when the private firms act as leaders and the public firm acts as the follower with that after privatization when they compete in a Cournot fashion. We obtain the following proposition.

**Proposition 6.** Consider the situation in which the public firm acts as a Stackelberg follower before privatization, while all firms engage in Cournot competition after privatization. The welfare before privatization is necessarily larger than that after privatization, irrespective of the number of private firms. That is, $W^V > W^{II} \forall n$.

**Proof.** By direct calculation, the following equation is satisfied: $W^V > W^{II} \iff (1+k)n^2+(1+k)(4+k)n+(2+k)^2 > (1+k)((3+k) + (4+k)n + n^2) \iff (2+k)^2 - (1+k)(3+k) = 1 > 0. \square$

Proposition 6 implies that the welfare before privatization is necessarily higher than that after privatization irrespective of the number of private firms. Combining Proposition 3 with Proposition 6, we can conclude that if firms engage in Stackelberg competition before privatization and they engage in Cournot competition after privatization, no privatization is preferable from the viewpoint of social welfare, irrespective of whether the public firm before privatization is a Stackelberg leader or a follower. Whenever the timing of output choice is sequential before privatization, privatizing the public firm necessarily deteriorates welfare. As suggested in Proposition 1, only when the timing of output choice is simultaneous there is scope for privatization to raise welfare, depending on the number of private firms.
However, it should be noted that the welfare comparison mentioned above presumes a slightly artificial situation because there is no justification for why the public firm changes from being a follower to being a simultaneous mover. Furthermore, like we pointed out in Subsection III.2, two different effects are mixed up when we consider privatization, that is, the changes in the firm’s objective and the timing of output choice. In the following analysis, without changing the timing of output choice, we only examine the effect of the change in the public firm’s objective by privatization, while the public firm remains a Stackelberg follower.

When we make a welfare comparison before and after privatization in the situation in which a public follower firm and private leader firms engage in Stackelberg competition, i.e., $W^V$ and $W^{VI}$, we obtain the following proposition.

Proposition 7. Consider the situation in which the public firm acts as a Stackelberg follower before and after privatization. If the number of private firms is less than 12, the welfare before privatization is larger than that after privatization irrespective of the size of the cost coefficient. That is, if $n = \{1, 2, \cdots, 11\}$, $W^V > W^{VI} \forall k > 0$.

Proof. The proof is in APPENDIX.

Proposition 7 suggests the possibility that even though the public firm remains a Stackelberg follower after privatization, privatization lowers the welfare. In particular, when the number of private firms is relatively small ($n = \{1, 2, \cdots, 11\}$), privatization leads to a decrease in welfare. The result is associated with the difference in advantageous moves before and after privatization. Before privatization, as shown in Proposition 5, there exists a second-mover advantage for the public firm, while after privatization, pure private competition starts and the public firm as a second mover loses its advantageous position because there exists a first-mover advantage in quantity competition between profit-maximizing firms. When there are relatively few private firms in the market, the welfare loss associated with the loss of second-mover advantage becomes large.

By the above various welfare comparisons, our results lead us to conclude as follows. When Stackelberg competition occurs in a mixed oligopoly, privatization brings about a decrease in welfare in many cases (especially when there are a few private firms in the market), irrespective of whether the public firm acts as a leader or a follower.

IV CONCLUDING REMARKS

In this paper, we revisited the work of De Fraja and Delbono (1989) and provided some additional results that extend the existing literature. We demonstrated the result that privatization cannot improve social
welfare in many cases. Among other things, we first showed that although there exists the possibility that privatization improves welfare when the public firm and private firms compete in a Cournot fashion, as already shown in De Fraja and Delbono (1989), if the number of private firms is small, privatization necessarily deteriorates welfare (Proposition 2). Second, we clarified that when the public firm acts as a Stackelberg leader before and also after privatization, privatization never improves welfare (Proposition 4). Third, we demonstrated that even when the public firm remains a Stackelberg follower before and after privatization, welfare deterioration is brought about by privatization if the number of private firms is relatively small (Proposition 7).

Our results suggest that the possibility of welfare improvement by privatization might be more restricted, unlike the existing literature which emphasizes the positive effect of privatization on welfare in mixed oligopolistic competition. Stated differently, our results are partially compatible with the fact that there are still many public firms that have not been privatized in various countries around the world, including in developed capitalist countries. The existence of public firms can be justified if transition to privatization brings about lower welfare in mixed oligopolistic markets.

Finally, we finish our paper by discussing possible extensions. We did not take the endogenization of the timing of output choice by firms into consideration. A firm’s choice of the timing of decision making is highly related to the firm’s objective. If the public firm has a second-mover advantage and the private firms have a first-mover advantage, the Stackelberg competition in which the public firm is a follower and private firms are the leader would be endogenously chosen. An extension of the mixed oligopoly framework to the endogenized timing of output choice is an issue for future research.
APPENDIX: Proof of Proposition 7

By direct calculation, the following relationship is satisfied: 
\[ W^V \geq W^{VI} \]
\[ \Leftrightarrow \frac{((1+k)n^2+(1+k)(4+k)n+(2+k)^2)}{(1+k)(1+\beta)^2} \geq \frac{(1+k)^2(2+k)^2n^2+(1+k)(2+k)^2(k^2+5k+2)n+(3+k)(1+k)^2+k^2)}{(2+k)^2(\beta+k(1+\beta))^2} \]
\[ \Leftrightarrow D(n; k) \equiv (1+k)(k^3+2k^2-k-1)n^2-(1+k)(2+k)(3k^2+8k+2)n-(2+k)^2((1+k)^2+k^2) \leq 0. \]
We denote the positive real-valued solution of the cubic equation, \( k^3+2k^2-k-1 = 0 \), as \( k^+ \), which is approximately 0.80194. When \( k = k^+ \), because \( k^3+2k^2-k-1 = 0 \) holds, \( D(n; k) \) is a linear function of \( n \) and the value of \( D(n; k^+) = -52.23n-128.71 < 0 \forall n \). Thus, when \( k = k^+ \), it is necessarily satisfied that \( W^V > W^{VI} \).
When \( k \neq k^+ \), that is, \( k^3+2k^2-k-1 \neq 0, D(n; k) = 0 \) is a quadratic equation of \( n \). We denote two solutions as \( n^+(k) \equiv \frac{(1+k)(2+k)(3k^2+8k+2)+\sqrt{(1+k)^2(2+k)^2(3k^2+8k+2)^2+4(1+k)(k^3+2k^2-k-1)(2+k)^2((1+k)^2+k^2)}}{2(1+k)(k^3+2k^2-k-1)} \)
and \( n^-(k) \equiv \frac{(1+k)(2+k)(3k^2+8k+2)-\sqrt{(1+k)^2(2+k)^2(3k^2+8k+2)^2+4(1+k)(k^3+2k^2-k-1)(2+k)^2((1+k)^2+k^2)}}{2(1+k)(k^3+2k^2-k-1)} \), where \( n^+(k) > n^-(k) \). Note that \( n^-(k) < 0 \) for all \( k > 0 \). When \( k \in (0,k^+) \), because \( k^3+2k^2-k-1 < 0 \) holds, \( D(n,k) \) is a quadratic function of \( n \) in which the coefficient of the square of \( n \) is negative. In this case, \( n^+(k) < 0 \) holds. Thus, when \( k \in (0,k^+) \), because \( n^-(k) < n^+(k) < 0 \) holds, \( D(n,k) \) becomes negative and \( W^V > W^{VI} \) is satisfied. On the other hand, when \( k > k^+ \), because \( k^3+2k^2-k-1 > 0 \) holds, \( D(n,k) \) is a quadratic function of \( n \) in which the coefficient of the square of \( n \) is positive. In this case, \( n^+(k) > 0 \) holds. Thus, by \( n^+ < 0 < n^- \), if \( n \in (0,n^+(k)) \), \( D(n;k) < 0 \) and as a result, \( W^V > W^{VI} \) are satisfied; if \( n > n^+(k) \), \( D(n;k) > 0 \) and \( W^V < W^{VI} \) are satisfied. Note that \( n^+(k) \) has the minimum value in the case of \( k > k^+ \). When \( k = k^{min} \approx 3.48883 \), this minimum value is \( n^{+min} \equiv n^+(k^{min}) \approx 11.216 \). Therefore, for all values of \( k \), if \( n \in \{1,2,\cdots,11\} \), it is necessarily satisfied that \( D(n;k) \) is negative, that is, \( W^V > W^{VI} \).
When \( n \geq 12 \), \( W^V > W^{VI} \) if \( n < n^+(k) \) and otherwise vice versa. \( \square \)
REFERENCES


TABLE 1: The equilibrium variables before and after privatization in Cournot competition

<table>
<thead>
<tr>
<th>I. Before privatization</th>
<th>II. After privatization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$a(k+1)$</td>
</tr>
<tr>
<td>$q$</td>
<td>$a$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$a(1 + k + kn)$</td>
</tr>
<tr>
<td>$p$</td>
<td>$ak(k+1)$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$\alpha^2 k^2(1+\frac{k}{2})$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\alpha^2(1+\frac{k}{2})$</td>
</tr>
<tr>
<td>$W$</td>
<td>$(1+k)^2 + nk = 1 + k(1 + \beta)$</td>
</tr>
</tbody>
</table>

III. before privatization  IV. after privatization

<table>
<thead>
<tr>
<th>$q_0$</th>
<th>$\frac{a}{t + \beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>$\frac{a}{t + \beta}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\frac{ak(k+1)}{t}$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$\frac{\alpha^2 k^2(1+\frac{k}{2})}{t^2}$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\frac{\alpha^2(1+\frac{k}{2})}{t^2}$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\frac{(1+k)^2 + nk = 1 + k(1 + \beta)}{2(1+\beta)^2 t^2}$</td>
</tr>
</tbody>
</table>

TABLE 2: The equilibrium variables before and after privatization in Stackelberg competition when a public firm is the leader

<table>
<thead>
<tr>
<th>III. before privatization</th>
<th>IV. after privatization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$\frac{at}{t + \beta}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\frac{a}{t + \beta}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\frac{ak(k+1) + nk}{t}$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$\frac{a^2 k^2 (1+\frac{k}{2})}{(t + \beta)^2}$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\frac{a^2(1+\frac{k}{2})}{(t + \beta)^2}$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\frac{(1+k)^2 + nk = 1 + k(1 + \beta)}{2(1+\beta)^2 t^2}$</td>
</tr>
</tbody>
</table>

TABLE 3: The equilibrium variables before and after privatization in Stackelberg competition when a public firm is the follower

<table>
<thead>
<tr>
<th>V. before privatization</th>
<th>VI. after privatization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$\frac{(2+k) a}{(1+k)(1+\beta)}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\frac{a}{1+\beta}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\frac{k(2+k) a}{(1+k)(1+\beta)}$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$\frac{2(1+k)^2 (1+\beta)^2}{k(5+3k) a^2}$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\frac{2(1+k)^2 (1+\beta)^2}{(k^2 + 4k + 2) a^2}$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\frac{(1+k)^2 + nk = 1 + k(1 + \beta)}{2(1+\beta)^2 t^2}$</td>
</tr>
</tbody>
</table>

(t $\equiv (1 + k)^2 + nk = 1 + k(1 + \beta)$, $\beta$ $\equiv 1 + k + n$)
FIGURE 1: $n^*(k) \equiv \frac{-k + \sqrt{k^2 + 4k(1+k)^3}}{2k} (> 0)$

FIGURE 2: $W^I$, $W^II$, and $W^III$