Donor Altruism and the Transfer Paradox in an Overlapping Generations Model

Kojun Hamada* and Mitsuyoshi Yanagihara†

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* Address: Faculty of Economics, Niigata University
8050 Ikarashi 2-no-cho, Nishi-ku, Niigata City 950-2181, Japan
Email address: khamada@econ.niigata-u.ac.jp
Tel. and fax: +81-25-262-6538

† Address: Graduate School of Economics, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan
Tel.: +81-52-789-5952
E-mail: yanagi@soec.nagoya-u.ac.jp
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Kojun Hamada
Faculty of Economics,
Niigata University

Mitsuyoshi Yanagihara†
Graduate School of Economics,
Nagoya University

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Abstract

This paper examines the transfer problem between two countries when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations model. We clarify that whether the transfer enriches the donor with strong altruism depends on the relative size of the marginal propensity to save between the donor and the recipient. We especially demonstrate that if the donor has a larger marginal propensity to save than the recipient, donor enrichment never occurs, irrespective of the degree of the donor’s altruism. Otherwise, donor enrichment occurs when the donor has a sufficiently high level of altruism.

Keywords: Altruism; Transfer paradox; Overlapping generations

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† Corresponding author.
Graduate School of Economics, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan
Tel.: +81-52-789-5952
E-mail: yanagi@socc.nagoya-u.ac.jp
1 Introduction

Why do countries provide foreign aid to other countries? One response is that it has long been accepted that a considerable amount of voluntary aid is implemented given the pure goodwill of the nationals concerned. That is, it is well recognized that transfers as an important form of foreign aid are at least partially supported by the altruism of the donor toward the recipient. For example, according to a survey conducted in 1983 by the Japanese Ministry of Foreign Affairs, among those respondents who evaluated Japan’s Official Development Assistance (ODA) program positively, the most common justification given was the stability and peace of developing countries (42.3%) followed by humanitarian obligations (32.7%). The results of this survey seemingly support the argument that the presence of altruistic feelings motivates Japan’s ODA. Conversely, another, and somewhat more realistic, reason for motivating a country to undertake international transfers arises from strategic considerations. That is, foreign aid is regarded as one means to achieve desirable diplomatic and/or economic policy goals for the assisting country itself. Among other things, Japan’s ODA White Paper (2010), which emphasizes the underlying philosophy of the Japanese foreign aid program, concludes that international transfers are not so much an act of charity from developed countries to developing countries, but rather a tool for the world community to pursue common interests. In short, we can explain the motivation for foreign aid from two perspectives, goodwill (altruism) and own benefit (selfishness), and note that which particular motivation dominates has likely changed over time.

If donor altruism is the main reason for undertaking transfers, introducing donor altruism into the existing transfer problem framework should be able to explain how a highly altruistic donor can improve utility by providing transfers to a recipient. Therefore, this paper examines the transfer problem between a donor and a recipient when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations (henceforth OLG) model and investigates the possibility for the transfer paradox to arise. To achieve this objective, we clarify whether the transfer paradox is likely to occur as the degree of donor altruism becomes high, as well as how this affects the welfare levels of both countries. In particular, we provide a response to the question of whether the motivation that encourages a donor to undertake an untied voluntary transfer is goodwill or benefit. More specifically, if the donor’s altruism enriches the donor in terms of utility, the transfer made by the donor country will be consistent with altruistic feelings

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1 See the Ministry of Foreign Affairs of Japan (1989, 2011).
by the donor nation and goodwill will be the main motivation for the transfer. Otherwise, altruism may not provide a reason for the donor to undertake the transfer. In this instance, we can recognize the transfer as merely another form of policy instrument for the donor government to achieve some economic or political objective. In other words, the benefit arising from the transfer will be the main motivation for the donor.

In a seminal paper on the transfer problem in a static framework, Samuelson (1952, 1954) showed that under free trade in a two-country framework, when the market equilibrium is Walrasian stable, neither a weak paradox, defined as the situation where both the donor and the recipient are enriched or immiserized (impoverished), nor a strong paradox, defined as the situation where the donor is enriched but the recipient is immiserized, can arise. In order for the transfer paradox to take place in a stable market equilibrium, several assumptions in Samuelson (1952, 1954) need to be modified. Since his seminal work, a voluminous static framework literature has shown how the transfer paradox arises by relaxing these assumptions. Various distortions have also been introduced into the static framework in order to explain the occurrence of the transfer paradox in a two-country model.

When individuals exhibit altruism toward other individuals, they obtain utility not only from their own consumption, but also from the utility levels of the other individuals. In this sense, we can regard altruism as a sort of externality, and this, like many other externalities, may cause economic distortion. In actuality, some studies have suggested that altruism itself could be a source of the transfer paradox, in particular through donor enrichment. For instance, Kemp and Shimomura (2002) employed a voluntary unrequited transfer model and showed that altruism could be the motivation for the donor to provide the transfer. Similarly, Lahiri and Raimondos-Møller (1999) developed a model where altruism is introduced to motivate the transfer and international trade is distorted through tariffs or quotas. These studies have not, however, fully proven that altruism itself brings about the transfer paradox. For example, in Kemp and Shimomura (2002), the transfer paradox is derived under a quite specific type of altruistic utility,

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2 The definitions of weak and strong paradox were first given by Yano (1983). We use these definitions throughout this analysis.

3 For instance, by extending the two-country model to a three-country model, Bhagwati, Brecher, and Hatta (1983) have shown the possibility of the transfer paradox when a bystander country other than the donor and recipient countries exists. Further, Bhagwati, Brecher, and Hatta (1985) have demonstrated that the transfer paradox can take place when exogenous distortions of trade barriers, such as tariffs and subsidies, or endogenous distortion, including lobbying and rent-seeking, prevail. The subsequent literature has considered the transfer paradox in a two-country model by introducing various distortions where free trade is hindered, for example, restrictions on the recipient country’s behavior in exchange for the transfer (Kemp and Kojima, 1985; Schweinberger, 1990; Lahiri and Raimondos-Møller, 1995), the administrative costs of the transfer (Kemp and Wong, 1993), and the transfer of production factors (Neary, 1995). For a concise survey of the transfer problem in a static model, see Brakman and Marrewijk (1998).
such that when the utility level of a country increases, the expenditure of the other country may also increase, despite the transfer. In Lahiri and Raimondos-Møller (1999), the transfer paradox arises not from altruism itself but from other distortions. Bearing these limitations in mind, Hamada (2012) has shown that even if the donor and/or recipient has altruistic utility, the transfer paradox never takes place in a simple two-country, two-commodity static model with no other distortion as long as the Walrasian stability of the equilibrium is guaranteed. This result implies that in a static framework, the benevolent assumption that the individuals of the donor country have altruistic intentions toward the recipient country cannot explain why the donor government voluntarily provides aid.

In contrast, because of capital accumulation and the movement of international capital in a dynamic framework, the transfer paradox can arise even with the Walrasian stability of an international capital market equilibrium under dynamic efficiency. Using a Diamond (1965) type, one-good, two-country OLG model, Galor and Polemarchakis (1987) first argued that a permanent lump-sum transfer can bring about the transfer paradox in a steady state being away from the golden rule. Subsequently, Haaparanta (1989) incorporated public debt into Galor and Polemarchakis (1987) and clarified that the transfer paradox can arise, even if the transfer is temporary, when financed by the issuance of public debt in the donor country and/or when the transfer is used for debt relief in the recipient country. Thus, when the temporary transfer relates to debt financing and/or relieving, it has the same long-run effect as a permanent lump-sum transfer. Tan (1998) also argued that any transfer from a rich to a poor country does not cause the transfer paradox in a steady state under dynamic efficiency. Utilizing a figure developed by Buiter (1981), Yanagihara (2006) explained graphically how the transfer paradox occurs, even if the economy is dynamically efficient. Cremers and Sen (2008) extended this analysis into the transition path converging to the steady state and showed that the results in Galor and Polemarchakis (1987) can also be applied to the transition path. Overall, in a dynamic framework, there is a possibility for the transfer paradox to occur in the dynamically efficient region.

In this paper, we clarify how introducing donor altruism into the utility of individuals in the donor country affects the possibility of the transfer paradox under the dynamic efficiency condition in an OLG framework. First, we explore whether the transfer paradox arises under the existence of altruism in a dynamic framework, though this cannot be acknowledged in a static framework. To clarify the difference in the results obtained with and without altruistic utility, we focus on the circumstance where the transfer paradox cannot transpire without altruism. Second, we investigate whether the donor’s altruism increases
the opportunity for donor enrichment.

We show that whether the transfer enriches the donor with strong altruism depends on the relative size of the marginal propensity to save between the donor and the recipient. We especially demonstrate that if the donor has a larger marginal propensity to save than the recipient, donor enrichment never occurs, irrespective of the degree of the donor’s altruism. Otherwise, donor enrichment occurs when the donor has sufficiently high altruism. These findings imply that the altruism of a donor toward a recipient does not necessarily explain the motivation to voluntarily provide a transfer. Further, if the donor has a larger marginal propensity to save than the recipient, the altruism of the donor’s individuals toward the recipient cannot justify the transfer made by the donor’s government. Rather, in this case, even if the donor’s altruism is sufficiently high, the transfer may cause the weak paradox such that both the donor and the recipient are immiserized, this being the Pareto-inferior outcome for both countries. On the other hand, if the donor has a relatively smaller marginal propensity to save, the donor’s altruism provides the reason why the transfer is made by the donor’s government. In this case, and in contrast with the former, if the donor’s altruism is sufficiently high, the transfer may account for a Pareto-improving outcome for both countries.

The remainder of the paper is organized as follows. Section 2 describes a one-sector OLG model wherein the individuals of the donor country have altruistic utility. Section 3 gives the welfare implications with regard to whether or when the transfer paradox takes place when there is donor altruism. Section 4 concludes the paper with some final remarks.

2 The model

Two countries comprise the world economy, a donor and the recipient of an international income transfer, indexed country $i = D$ and $R$, respectively. These two countries are identical except for their time preferences and the altruism of individuals. Capital is fully mobile between the countries, however, goods and labor are immobile. The (gross) growth rates of the population in both countries are exogenously given, identical, and constant over time: $1 + n \geq 1$. 
2.1 Individuals

Individuals live for two periods. In each period, both countries are populated by two generations, the young who inelastically supply labor for one unit of time and earn wages, and the old who retire. We assume that the individuals of the donor country exhibit altruism toward the individuals of the recipient country, while the individuals of the recipient country display no altruism. $u^i_t = u^i(c^i_t, d^i_{t+1})$ denotes the (sub-) utility function that the individuals of country $i$ born in period $t$ (we refer to this as generation $t$) obtain from their own consumption when young, $c^i_t$, and when old, $d^i_{t+1}$. In the model, the altruism of the individuals in country $D$ is described as follows. We assume that the total utility of the donor’s individuals depends on both $u^D_t$ and $u^R_t$, that is, the selfish utility and the altruistic utility obtained from the utility of the recipient’s individuals. Thus, generation $t$ in country $D$ with altruism has the total utility function denoted by $U^D_t \equiv U^D(u^D_t, u^R_t) = U^D(u^D_t(c^D_t, d^D_t), u^R_t(c^R_t, d^R_t))$. Alternatively, as the individuals in country $R$ do not display any altruism, their total utility is denoted by $u^R_t = u^R_t(c^R_t, d^R_t)$. $u^i(\cdot)$ and $U^D(\cdot)$ are assumed to be twice-differentiable, increasing, and quasi-concave in $(c^i_t, d^i_{t+1})$, i.e., $\frac{\partial u^i}{\partial c^i_t} = u^i_c > 0$, $\frac{\partial u^i}{\partial d^i_{t+1}} = u^i_d > 0$, and $u^i_{cc}u^i_{dd} - (u^i_{cd})^2 > 0$. Moreover, we assume that $\frac{\partial U^D_D}{\partial c^D} > U^D_R \equiv \frac{\partial U^D}{\partial c^R} \geq 0$, which implies that the effect of self-utility on total utility is always larger than that of altruistic utility. The larger $U^D_R$ grows, the stronger the altruistic feelings become.

The budget constraints of generation $t$ in their young and old periods in country $i = D, R$ are, respectively:

$$c^i_t + s^i_t = w_t + T^i$$

and

$$d^i_{t+1} = r_{t+1}s^i_t,$$

where $r_{t+1}$, $w_t$, and $s_t$ are the gross interest rate in period $t + 1$ (equal to the gross return on savings in period $t$), wages in period $t$, and savings in period $t$, respectively. The net income in period $t$ consists of the wage and a permanent international lump-sum transfer $T^i$. We convert these two budget constraints into the following lifetime budget constraint:

$$c^i_t + \frac{1}{r_{t+1}}d^i_{t+1} = w_t + T^i.$$

Individuals in both countries maximize their total utilities subject to their lifetime budget constraints (2). The utility maximization problem for the individuals of country $i$ facing the lifetime budget con-
straint (2) can be formulated as:

$$\max_{\{c_t^D, d_t^D\}} U^D(c_t^D, d_t^D(w_t, r_{t+1})), \quad \text{s.t.} \quad c_t^D + \frac{1}{r_{t+1}}d_t^D = w_t - T, \quad (3)$$

$$\max_{\{c_t^R, d_t^R\}} u^R(c_t^R, d_t^R(w_t, r_{t+1})), \quad \text{s.t.} \quad c_t^R + \frac{1}{r_{t+1}}d_t^R = w_t + T, \quad (4)$$

where $T \equiv T^R = -T^D(>0)$ defines the permanent transfer from the donor to the recipient. It should be noted that when the individuals of country $D$ determine their levels of consumption, they only care about their own self-utility $u^D$ and take the altruistic utility $u^R$ as given. This is because they cannot control the consumption levels of individuals in another country. That is, $\arg \max U^D = \arg \max u^D$.

The first-order condition gives the optimal consumption bundle, $(c_t^D(w_t + T', r_{t+1}), d_t^D(w_t + T', r_{t+1}))$ and the savings function, $s_t^D(w_t + T', r_{t+1}) \equiv \frac{d_t^D(w_t + T', r_{t+1})}{r_{t+1}}$. As in Haaparanta (1989) and Yanagihara (1998, 2006), we assume that the savings function is increasing in both the wage and interest rate, that is, $\frac{\partial s_t^D}{\partial w_t} > 0$ and $\frac{\partial s_t^D}{\partial r_{t+1}} > 0$.

Substituting consumption and savings into the total utility function, we obtain the indirect utility function:

$$V^D(w_t, r_{t+1}; T) \equiv U^D\left(c_t^D(w_t - T, r_{t+1}), d_t^D(w_t - T, r_{t+1})\right), \quad u^R\left(c_t^R(w_t + T, r_{t+1}), d_t^R(w_t + T, r_{t+1})\right), \quad (5)$$

$$V^R(w_t, r_{t+1}; T) \equiv u^R\left(c_t^R(w_t + T, r_{t+1}), d_t^R(w_t + T, r_{t+1})\right). \quad (6)$$

Given $\frac{\partial c_t^D}{\partial w_t} + \frac{\partial c_t^R}{\partial w_t} = u_t^D > 0$ and $\frac{\partial c_t^R}{\partial r_{t+1}} > 0$, the indirect utility functions of both countries have the following properties:

$$V^D_t = U^D_t u_t^D + U^R_t s_t^D u_t^R, \quad V^R_t = U^D_t s_t^R u_t^D + U^R_t s_t^R u_t^R, \quad (7)$$

$$V^D_t = -U^D_t u_t^D + U^R_t u_t^R, \quad V^R_t = s_t^R u_t^D, \quad \text{and} \quad V^D_t = u_t^R. \quad (8)$$

For simplicity, the marginal self-utility of consumption in the young period can be normalized to unity ($u_t^D = 1$), so that $u_t^R = \frac{1}{r_{t+1}}$. Likewise, the marginal effect of self-utility on the donor’s total utility can be normalized to unity. Moreover, we assume that when evaluating the neighborhood of the equilibrium,

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4 We define $V_t^\ell = \frac{\partial V_t'}{\partial w'}$, $V_t^R = \frac{\partial V_t'}{\partial r_{t+1}}$, and $V_t^R = \frac{\partial V_t'}{\partial T'}$. 

7
the marginal effect of the altruistic utility is less than unity: $U_D^{\alpha} = 1$ and $U_R^{\alpha} = \alpha \in [0, 1)$. The value of $\alpha$ indicates the degree of altruism of individuals in the donor country toward individuals in the recipient country. Thus, (7) and (8) are arranged as follows:

$$
V_w^D = 1 + \alpha, \quad V_r^D = \frac{s_r^D + \alpha s_r^R}{r_{t+1}}, \quad V_f^D = -(1 - \alpha), \quad (9)
$$

$$
V_w^R = 1, \quad V_r^R = \frac{s_r^R}{r_{t+1}}, \quad \text{and} \quad V_f^R = 1. \quad (10)
$$

### 2.2 Firms

Firms in both countries produce output under perfect competition. The production function exhibits constant returns to scale in capital and labor, independent of time, and is identical in both countries.

We denote the per capita production function by $f(k_i^t)$, where $k_i^t$ represents per capita capital in country $i$ in period $t$. This is assumed to satisfy the following conditions: (i) $f(\cdot)$ is continuously differentiable and (ii) $f(k_i^t) > 0$, $f'(k_i^t) > 0$, and $f''(k_i^t) < 0$ for all $k_i^t > 0$. Moreover, we assume the Inada conditions: (iii) $f(0) = 0$ and (iv) $\lim_{k_i \to 0} f'(k_i) = \infty$ and $\lim_{k_i \to \infty} f'(k_i) = 0$. For simplicity, we assume that capital does not depreciate over time.

Firms maximize their profit in per capita terms denoted by $\pi(k_i^t) \equiv f(k_i^t) - r_t k_i^t - w_t$. Profit maximization requires the equivalence of marginal productivity and the price of each input such that:

$$
f'(k_i^t) = r_t \quad \text{and} \quad f(k_i^t) - f'(k_i^t) k_i^t = w_t. \quad (11)
$$

From the first equation of (11), we obtain the capital demand function represented by $k_i^t(r_t)$, where $k_i^t(r_t) = 1/f'' < 0$. This implies that (per capita) capital demand is decreasing in the interest rate $r_t$. Similarly, from the second equation of (11), $w_i^t(r_t) = -k_t < 0$ is obtained.

### 2.3 Equilibrium

We consider a world capital market equilibrium in period $t$, which requires the sum of the per capita savings of the young generation in both countries in period $t$ to equal the sum of per capita capital demand in the subsequent period $t + 1$. As capital is perfectly mobile, the interest rates in both countries become the same, so that $k_{t+1}^D = k_{t+1}^R$ holds through factor price equalization. Therefore, given $w_t$, or
equivalently \( k_t \), the capital market equilibrium in period \( t \) is expressed as follows:

\[
2(1+n)k_{t+1}(r_{t+1}) = s^D(w_t(r_t) - T, r_{t+1}) + s^R(w_t(r_t) + T, r_{t+1}).
\] (12)

Define the excess demand in the world capital market as 
\[ D(w_t, r_{t+1}) \equiv 2(1+n)k_{t+1}(r_{t+1}) - s^D(w_t - T, r_{t+1}) - s^R(w_t + T, r_{t+1}). \]

Then, under the assumption that savings are increasing in the interest rate,

\[
\Delta_t = \frac{\partial D(w_t, r_{t+1})}{\partial r_t} = 2(1+n)k'_{t+1}(r_{t+1}) - s^D - s^R < 0
\] (13)

holds. Therefore, the Walrasian stability condition is satisfied in the capital market equilibrium in each period.

### 3 Donor altruism and the transfer paradox

We focus on the steady-state analysis in order to investigate the effect of the transfer on the donor’s and recipient’s welfare when the donor has altruistic utility. As usually assumed when analyzing the steady state, following Sibert (1985), we limit our analysis to the case in which the economy is dynamically efficient, that is, \( r_t > 1 + n \) for all \( t \).

#### 3.1 Savings and the interest rate

From (12), we immediately obtain the equilibrium condition of the world capital market in the steady state as follows:

\[
2(1+n)k(r) = s^D(w(r) - T, r) + s^R(w(r) + T, r).
\] (14)

In order for the economy to converge monotonically to the steady-state equilibrium, we assume the following dynamic stability condition:

\[
\Gamma \equiv \frac{dD(w(r), r)}{d r} = 2(1+n)k'(r) - s^D - s^R - (s^D + s^R)w'(r) = \Delta + (s^D + s^R)k < 0,
\] (15)

---

\(^5\) Henceforth, the variables with no subscript \( t \) represent those in the steady state.
where $\Delta \equiv 2(1 + n)k'(r) - s^D_r - s^R_r$, and $\Delta < 0$ under the Walrasian stability condition. It should be noted that the difference between $\Gamma$ and $\Delta$ is $(s^D_r + s^R_r)k$, as obviously obtained from comparing (13) with (15). This difference comes from the fact that the change in the interest rate affects wages through the change in factor demand by firms and this brings about the long-run effect through the change in capital accumulation.

By totally differentiating (14), we obtain the effect of the transfer on the interest rate as follows:

$$\Gamma dr = (s^R_w - s^D_w) dT. \quad (16)$$

With respect to the difference in the marginal propensity to save between the donor and the recipient, we obtain the following lemma. All proofs are in the Appendices.

**Lemma 1.** Suppose that the time preference of the recipient is higher (lower) than that of the donor. In the steady state,

(i) $s^R < (1 + n)k < s^D$ ($s^R > (1 + n)k > s^D$) and

(ii) the transfer increases (decreases) the interest rate.

It should be noted that Lemma 1 holds irrespective of the degree of donor altruism. Part (i) of this lemma implies that the individuals in the country with higher time preference have smaller savings and they therefore need more capital to invest from the other country. In contrast, the individuals in the country with lower time preference have larger savings and can therefore supply their capital to the other country, which is in need of capital. Therefore, the difference in the marginal propensity to save between the donor and the recipient, $(s^R_w - s^D_w)$, determines the direction of international capital movement between the two countries. If $s^R_w < s^D_w$, the donor is the lender of capital and the recipient is the borrower of capital; otherwise, vice versa.

Part (ii) of Lemma 1 implies the following. As the recipient with higher time preference has a lower marginal propensity to save than the donor, the donor becomes the lender of capital and the recipient becomes the borrower of capital. Although the savings of the donor decrease and those of the recipient increase given the transfer from the donor to the recipient, total world capital decreases. As a result, if the marginal propensity to save is higher in the donor, the transfer from the donor to the recipient brings about an increase in the interest rate; otherwise, vice versa.
3.2 The effect of the transfer on welfare

We now proceed to investigate the effect on welfare in the steady state, as defined by the representative individual’s utility in the country. By totally differentiating the indirect utility functions in the steady state and substituting (A.3), (10), and \( dw = -kdr \) into them, we obtain the following equations.

\[
dV_D = V_D w dw + V_D^r dr + V_T^D dT = \left( -(1 + \alpha)k + \frac{s^D + \alpha s^R}{r} \right) dr - (1 - \alpha) dT, \tag{17}
\]

\[
dV_R = V_R w dw + V_R^r dr + V_T^R dT = \left( -k + \frac{s^R}{r} \right) dr + dT. \tag{18}
\]

From (17) and (18), we find that both the donor’s and the recipient’s welfare are affected by the changes in both the interest rate and the transfer, as shown in existing studies. By substituting (16) into (17) and (18), we obtain the effects on welfare in both the donor and recipient countries as follows:

\[
dV_D = \left[ \left( s^D - rk \right) + \left( s^R - rk \right) \alpha \right] \frac{s^R - s^D}{rT} - (1 - \alpha) T; \tag{19}
\]

\[
dV_R = \left[ \left( s^R - rk \right) \frac{s^R - s^D}{rT} + 1 \right] dT. \tag{20}
\]

The total effect of the transfer on welfare is divided into two parts, a direct effect and an indirect effect. The direct effect is the income effect brought about by the change in the income level of both countries by the transfer itself. This effect is necessarily positive for the recipient country and negative for the donor country, and is shown by the second term in both (19) and (20). The indirect effect is what we call an intertemporal terms-of-trade effect and is shown by the first term in both (19) and (20). The indirect effect is the result of the following process: changes in the level of income given the transfer alter the pattern of consumption and savings, which in turn change the level of world capital, unless the marginal propensities to save in both countries are the same. The change in world capital then brings about a permanent change in the interest rate.

Noting that \( s^i - rk = -(r - (1 + n))k + (s^i - (1 + n)k) \), the indirect intertemporal terms-of-trade effect...
can be further decomposed into two effects, the capital accumulation effect and the capital movement effect. For example, in (20), the former is shown by 

\[-(r-(1+n))k\frac{s^R-s^D}{\theta},\]

because \(-(r-(1+n))k\) indicates how the level of capital moves away from the level in the golden rule: when the economy is dynamically efficient, as assumed, and capital accumulation is promoted, the increase in production given the increase in accumulated capital brings about an increase in the utility of both countries. Alternatively, the capital movement effect is shown by

\((s^R-(1+n)k)\frac{s^R-s^D}{\theta},\)

where \((s^R-(1+n)k)\) represents the direction and amount of capital movement: the country in which this term is positive (negative) becomes the lender (borrower) of capital. If world capital is accumulated and, as a result, the interest rate decreases, the capital movement effect decreases the welfare of the capital lender and increases that of the capital borrower. It should be noted here that the capital accumulation effect works in the same direction as the level of welfare in both countries, whereas the capital movement effect works in the opposite direction to the level of welfare in both countries. Therefore, the difference in the impact of the capital movement effect between the donor and the recipient provides one source of the transfer paradox.

As an illustrative example, consider the case in which the donor has a larger marginal propensity to save than the recipient, i.e., \(s^R_w < s^D_w\). Since \(\frac{s^R-s^D}{\theta} > 0\), the sign of the recipient’s indirect effect of (20) corresponds to the sign of \((s^R-(1+n)k)\frac{s^R-s^D}{\theta}\). The first term, \(-(r-(1+n))k\), which determines the sign of the capital accumulation effect, is always negative because in a dynamically efficient situation the transfer decreases world per capita capital and increases the interest rate. The second term, \((s^R-(1+n)k)\), which determines the sign of the capital movement effect, becomes negative because the recipient country is a capital borrower given Lemma 1. The intertemporal terms-of-trade effect then consists of both the above effects.

As shown in (19) and (20), the donor’s indirect effect depends not only on \((s^D-rk)\) but also on \((s^R-rk)\) because of the existence of altruism, while the recipient’s indirect effect depends only on \((s^i-rk)\). \((s^i-rk), i = D, R\) is positive (negative) if and only if the capital movement effect is larger (smaller) than the capital accumulation effect. Therefore, if the capital accumulation effect is sufficiently larger than the capital movement effect, the indirect effects for both countries become negative. On the contrary, if the capital accumulation effect is sufficiently small, it is likely that the directions of the donor’s and the recipient’s indirect effects differ. Thus, \((s^i-rk)\) determines the sign of the indirect effect.

As the degree of donor altruism becomes higher, the direct income effect, which is negative for the donor country, becomes smaller. In particular, if \(\alpha = 1\) at the most extreme, it is nullified. However,
the indirect terms-of-trade effect remains, even though the degree of donor altruism increases up to the extreme. Therefore, the indirect effect triggers the occurrence of the transfer paradox.

3.3 The transfer paradox

We are now in a position to clarify the conditions for the transfer paradox to occur in the steady state. First, we examine the indirect effect of the transfer on welfare. The following two lemmas summarize the sign of the donor’s and the recipient’s indirect effects.

Lemma 2. In the steady-state equilibrium,
(i) if \( s_w^R < s_w^D \) and \( s^D \leq rk \), the donor’s indirect effect is always negative irrespective of the degree of donor altruism;
(ii) if \( s_w^R < s_w^D \) and \( s^D > rk \), there exists a threshold of \( \alpha, \alpha_0 \), such that the donor’s indirect effect is positive if \( \alpha < \alpha_0 \), and negative if \( \alpha > \alpha_0 \);
(iii) if \( s_w^R > s_w^D \), the donor’s indirect effect is always positive irrespective of the degree of donor altruism.

Lemma 3. In the steady-state equilibrium, the recipient’s indirect effect is independent of the degree of donor altruism. Moreover,
(i) if \( s_w^R < s_w^D \) or if \( s_w^R > s_w^D \) and \( s^R > rk \), the recipient’s indirect effect is negative;
(ii) if \( s_w^R > s_w^D \) and \( s^R \leq rk \), the recipient’s indirect effect is positive.

As shown in (19), when \( s_w^R < s_w^D \), the donor’s indirect effect becomes negative when the capital accumulation effect dominates the capital movement effect. Consider when the donor’s marginal propensity to save is higher than that of the recipient, but not so high that \( s^D \leq rk \) holds (that is, \( s^D \), which equates to capital lending to the recipient, is not so large). Then, the transfer from the donor to the recipient decreases world total savings and increases the interest rate. However, this increase in the interest rate does not bring about a sufficiently large positive capital movement effect to dominate the negative capital accumulation effect. In addition, under dynamic efficiency, \( r > 1 + n, s^R < rk \) necessarily holds. As a result, the donor’s indirect effect becomes negative. This case corresponds to Lemma 2(i). If the donor has such a large marginal propensity to save that the capital lending is sufficiently large, then there is a possibility for the positive capital movement effect to dominate the negative capital accumulation effect. Even in this case, however, as the altruistic donor considers the negative impact of the indirect effect on the recipient, the greater donor altruism weakens the positive capital movement effect on the donor.
implies that a threshold for the degree of donor altruism exists, such that the sign of the donor’s indirect effect reverses. Lemma 2(ii) states this case. Contrary to the above two cases, Lemma 2(iii) insists that if the donor has a smaller marginal propensity to save than the recipient, the donor’s indirect effect is necessarily positive. In this case, the transfer increases world total savings and lowers the interest rate. The former brings about positive capital accumulation effects in both countries and the latter imposes a positive capital movement effect on the capital lender, that is, the donor. In fact, as under dynamic efficiency \( s^D + s^R = 2(1 + n)k < 2rk \) holds, the donor’s indirect effect is necessarily positive.

In contrast to the effect on the donor, because the recipient is not at all altruistic towards the donor, it is sufficient only to investigate the indirect effect on the recipient’s own utility. As shown in (20), the recipient’s indirect effect depends only on \((s^R - rk)\). Thus, the configuration of \(s^R\) and \(rk\) completely determines the sign of the recipient’s indirect effect. Lemma 3 implies that if the donor has a larger marginal propensity to save than the recipient, the transfer decreases world capital, which represents the negative capital accumulation effect, and increases the interest rate, which represents the negative capital movement effect on the recipient in the steady state, as with Lemma 2(i) and (ii). In fact, \(s^R < (1 + n)k < rk\) holds under dynamic efficiency regardless of the presence of donor altruism. This result lies in contrast to that in Lemma 2(iii). If, on the other hand, the recipient has a larger marginal propensity to save than the donor, even though the capital accumulation effect is positive, the capital movement effect on the recipient becomes negative. Therefore, as has already been pointed out regarding the existing literature, Lemma 2 and Lemma 3 confirm that the direction of the indirect effect is determined by the configuration of the above two countervailing effects.

Now we examine how the altruism of the donor has an effect on the occurrence of the transfer paradox. As the recipient’s welfare does not depend on the degree of donor altruism, we investigate only the possibility for donor enrichment, in the situation where no kind of transfer paradox — strong or weak — occurs without the donor’s altruism. We present the following proposition concerning donor enrichment.
Proposition 1. Suppose the situation where no transfer paradox occurs without the donor’s altruism. In the steady-state equilibrium,

(i) when the donor has a larger marginal propensity to save than the recipient, donor enrichment never occurs, irrespective of the degree of donor altruism;
(ii) when the donor has a smaller marginal propensity to save than the recipient, donor enrichment occurs if the donor displays sufficient altruism.

Proposition 1 specifies the condition for donor enrichment to occur in the normal situation, where no transfer paradox occurs without donor altruism. Proposition 1(i) implies that altruism never contributes to bringing about paradoxical effects on the donor in the normal situation. As shown in (19), the direct effect is necessarily negative, even though it is weakened by the introduction of altruism. In addition, because \( (s^D - rk) \frac{r^G}{n} < 0 \) holds in the normal situation, and \( s^R < (1 + n)k < rk \) holds given dynamic efficiency, the indirect effect is also negative. It should be noted that when \( s^R_w < s^D_w \), altruism only magnifies the negative indirect effect and never compensates for the negative direct effect on the donor itself. In sum, when the donor has a larger saving propensity, the transfer always worsens donor welfare, irrespective of the existence of altruism.

In contrast, Proposition 1(ii) asserts that when the donor has a smaller marginal propensity to save than the recipient, donor enrichment occurs if the donor has sufficient altruism toward the recipient. This can be explained as follows. As shown in Lemma 2(iii), when \( s^R_w > s^D_w \), the donor’s indirect effect is always positive irrespective of the degree of donor altruism, whereas the direct effect is negative. When \( \alpha \) approaches unity, although the donor’s negative direct effect is nullified, the positive indirect effect still exists. As a result, when the donor has a smaller saving propensity, donor altruism leads to improving donor welfare when the degree of donor altruism is sufficiently large. Intuitively, the increase in world capital given the transfer decreases the interest rate, which in turn decreases the return for capital borrowing from the recipient, and this indirect effect dominates the negative direct effect, which becomes weaker as the degree of altruism becomes larger.

The above result suggests that whether the donor’s altruism can explain the motivation for the transfer depends on the relative size of the marginal propensity to save between the donor and the recipient. If the donor has a larger marginal propensity to save than the recipient, which we might often expect in real-world situations, the donor’s altruism cannot explain the motivation for the donor to undertake the transfer because the transfer never enriches the donor. In contrast, if the donor has a smaller marginal
propensity to save than the recipient, the result that the good you do for others is good you do yourself can be acknowledged and the donor’s altruism then brings about an improvement in its own welfare.

Next, we extend the argument in the above normal situation to a general situation where the transfer paradox might occur even if there is no donor altruism. Our aim in this extension is to examine how the donor’s altruism has an effect on the occurrence of a strong transfer paradox. Generally speaking, when there is no donor altruism, as supposed by Galor and Polemarchakis (1987), the following four cases can be considered: (a) No transfer paradox occurs, i.e., neither the donor’s enrichment nor the recipient’s immiserization occurs; (b) Only a weak transfer paradox in which the donor is enriched occurs; (c) Another weak transfer paradox in which the recipient is immiserized occurs; (d) The strong transfer paradox occurs, i.e., both the donor’s enrichment and the recipient’s immiserization occur at the same time. Although we have only dealt with Case (a) in Proposition 1, we now extend the analysis to cover all four cases.

When we examine the general situation including Cases (a)–(d), we provide the following proposition about the possibility of the donor’s paradoxical enrichment.

**Proposition 2.**

(i) Suppose in the steady-state equilibrium that the donor has a larger marginal propensity to save than the recipient. If the donor has sufficiently high altruism, the transfer never enriches the donor. That is, when \( s^R_w < s^D_w \), \( \frac{dV_D}{dT} < 0 \) is likely to hold as \( \alpha \) approaches unity.

(ii) Suppose in the steady-state equilibrium that the donor has a smaller marginal propensity to save than the recipient. If the donor has sufficiently high altruism, the transfer necessarily enriches the donor. That is, when \( s^R_w > s^D_w \), \( \frac{dV_D}{dT} > 0 \) is likely to hold as \( \alpha \) approaches unity.

Proposition 2 implies that whether the transfer enriches the donor with strong altruism depends on the relative size of the marginal propensity to save between the donor and the recipient. Proposition 2(i) shows that if the donor has a large marginal propensity to save, the transfer necessarily reduces the donor’s own welfare, even when the donor has sufficiently large altruism towards the recipient. This result seems to be counterintuitive at first glance, as it is widely believed that one of the motivations for international transfers is the principle of reciprocity. However, this result suggests that such a reciprocal view, that charity brings its own reward, does not necessarily hold. In practice, when the transfer is made from the developed country as a donor, which is usually a higher savings country, to the developing
country as a recipient, which is usually a lower savings country, the developed country as a donor cannot enjoy the benefit of its own welfare improvement through the transfer.

The intuition behind this result is essentially the same as that in Proposition 1(i). As $\alpha$ approaches unity, the welfare of the donor becomes equal to world welfare, consisting of the sum of the donor’s and the recipient’s welfare. In such a situation, the direct income effect vanishes and only the indirect effect remains. In the remaining indirect effect, although the capital movement effect also vanishes, the capital accumulation effect remains. As a result, as the transfer decreases world capital in the steady state under dynamic efficiency, the donor’s welfare necessarily decreases.

In contrast, if the donor has a smaller marginal propensity to save than the recipient, a different result can be derived. Proposition 2(ii) insists that when the donor has a smaller marginal propensity to save, if the donor is highly altruistic, the transfer necessarily improves the donor’s own welfare. In this case, the result appears as expected and the principle of reciprocity can explain the motivation for an international transfer by the donor. Stated differently, the old proverb that charity brings its own reward holds for the donor that makes smaller savings, that is, borrows capital. The basic logic of Proposition 2 is quite natural: as long as more world capital is accumulated by the transfer from the donor with smaller savings and high altruism to the recipient with larger savings, the donor can obtain a benefit from capital accumulation that exceeds the loss from the reduction in income caused by the transfer. Therefore, although greater donor altruism can be compatible with donor enrichment when the donor’s marginal propensity to save is lower than that of the recipient, the donor’s greater altruism by itself cannot cause donor enrichment in the opposite case. From the above results, we conclude that in a dynamic framework, the relative size of the marginal propensity to save between a donor and a recipient determines whether the transfer enriches the donor. This potentially provides one of the motivations for a donor to undertake a voluntary transfer.

In order to delineate the results shown in Proposition 2, we present Table 1 by classifying the effects of transfers without donor altruism into the above four cases. From Table 1, we can confirm that when $s_R < s_D$, in most cases the donor becomes worse off, except where $\alpha < \alpha_1$ in Case (d). As shown in Table 1, this implies that the strong transfer paradox never occurs if the degree of donor altruism is sufficiently large when $s_R < s_D$. In other words, the transfer necessarily brings about a decrease in the welfare of either the donor or the recipient. In contrast, when $s_R > s_D$, the donor becomes better off in most cases except where $\alpha < \alpha_1$ in Case (a). When the recipient has a larger marginal propensity to save than the
donor, there exists the possibility of a strong transfer paradox, and the weak paradox under which welfare
in both countries deteriorates can never occur. It should be noted, however, that as shown in Case (d), the
strong transfer paradox occurs in the circumstance with donor altruism only if it occurs without donor
altruism in the first instance. Therefore, the strength of the donor’s altruism itself cannot provide any
rationale for the strong paradox.

\[
\begin{array}{cccc}
  s^R_w < s^D_w & s^R_w > s^D_w \\
  \text{donor} & - & - & \text{if } \alpha < \alpha_1 \\
  & & + & \text{if } \alpha > \alpha_1 \\
  \text{recipient} & + & + & \\
\end{array}
\]

\[
\begin{array}{cccc}
  s^R_w < s^D_w & s^R_w > s^D_w \\
  \text{donor} & N/A & + & \\
  \text{recipient} & + & + & \\
\end{array}
\]

(a) \(\left. \frac{dv^D}{dt} \right|_{\alpha=0}, \frac{dv^R}{dt} \right|_{\alpha=0} = (-, +)\)

(b) \(\left. \frac{dv^D}{dt} \right|_{\alpha=0}, \frac{dv^R}{dt} \right|_{\alpha=0} = (+, +)\)

\[
\begin{array}{cccc}
  s^R_w < s^D_w & s^R_w > s^D_w \\
  \text{donor} & - & N/A & \\
  \text{recipient} & - & - & \\
\end{array}
\]

(c) \(\left. \frac{dv^D}{dt} \right|_{\alpha=0}, \frac{dv^R}{dt} \right|_{\alpha=0} = (-, -)\)

(d) \(\left. \frac{dv^D}{dt} \right|_{\alpha=0}, \frac{dv^R}{dt} \right|_{\alpha=0} = (+, -)\)

Table 1: The effect of the transfer on welfare

Define \(\alpha_1 \equiv -\frac{\Lambda^D}{\Lambda^R}\). When \(s^R_w < s^D_w\), Case (b) cannot occur and when \(s^R_w > s^D_w\), Case (c) cannot occur.

Our result lies in stark contrast to that obtained in the static framework, in which no transfer paradox
occurs, even when a donor exhibits altruism towards its recipient. In the static framework, even though
the direct income effect of the transfer countervails the indirect terms-of-trade effect, the latter can never
dominate the former under Walrasian stability. As long as the Walrasian stability condition holds, the
total effect of the transfer necessarily brings about a decrease in the donor’s welfare and an increase in
the recipient’s welfare in the static setting. In contrast, in our dynamic framework, even when the world
capital market equilibrium is Walrasian stable in every period, we have shown that the indirect effect,
including that for capital accumulation, could exceed the direct effect to bring about the transfer paradox
in the steady state. In particular, we demonstrate that whether the degree of donor altruism raises the
possibility of the donor’s enrichment paradox depends on the difference in the marginal propensity to
save between the donor and the recipient. This discrepancy between the static and dynamic models is
mainly attributed to the intertemporal terms-of-trade effect, although whether the transfer paradox arises
when there is donor altruism depends on the relative size of the marginal propensity to save in both countries.

4 Concluding remarks

This paper demonstrated that the occurrence of the transfer paradox for a donor country depends on the relative size of the marginal propensity to save in the donor and the recipient countries. We showed that if the donor has a larger marginal propensity to save than the recipient, the donor’s enrichment never occurs, irrespective of the degree of donor altruism; otherwise, the donor’s enrichment occurs when the donor has sufficiently large altruism. Furthermore, we suggested that when the degree of donor altruism is sufficiently large, if the donor has a large marginal propensity to save, a Pareto-inferior but no Pareto-improving result is likely to occur. In contrast, given sufficiently large donor altruism, if the donor has a small marginal propensity to save, a Pareto-improving but no Pareto-inferior result is likely to occur. Our result thus suggests that if the donor is a higher saving country compared with the recipient, transfer on the basis of goodwill may bring about a Pareto-inferior outcome for both countries. Therefore, if the donor is a high saving country, the donor’s altruism cannot explain the motivation for transfer.

Finally, we conclude this paper with some possible future extensions. To start with, we limit our analysis to a model in which only the donor exhibits altruism toward a recipient. As a direct extension, and even though understandably complex, this analysis should be extended to a generalized model in which both the donor and the recipient exhibit altruism toward each other. However, even were we to extend the model to this case, we expect the fundamental results in this paper to hold. Nonetheless, through this extension we could obtain a more fruitful conclusion about the impact of the different degrees of altruism on donors and recipients. As another extension, we focus only on the steady-state equilibrium, and so future analysis could consider the transition path. Either of these extensions would yield useful insights into the relationship between altruistic behavior and international transfer activities.
Appendix

A The proof of Lemma 1

(i) In the steady state, the time preference of the recipient is higher (lower) than that of the donor, if and only if the marginal propensity to save of the recipient is lower (higher) than that of the donor. That is, when $s_w^R < s_w^D$ holds for all $t$, the steady-state savings of the recipient become necessarily smaller than those of the donor, i.e., $s^R < s^D$, irrespective of the initial capital level. By combining $s^R < s^D$ with (14), we obtain $s^R < (1+n)k < s^D$. On the other hand, when $s_w^R > s_w^D$ for all $t$, the opposite result holds. (ii) By (16), we immediately obtain that $\frac{ds^R}{dt} \geq 0$ if and only if $s_w^R \leq s_w^D$. □

B The proof of Lemma 2

Denote the donor’s indirect effect as $X(\alpha) \frac{\alpha^R - \alpha^D}{\Gamma}$, where $X(\alpha) \equiv (s^R - rk) \alpha + (s^D - rk)$. Because of the dynamic stability condition, $\Gamma < 0$, $s_w^R \leq s_w^D$ if and only if $\frac{\alpha^R - \alpha^D}{\Gamma} \geq 0$. The sign of the donor’s indirect effect is determined by the sign of $X(\alpha)$. From Lemma 1, $s_w^R \leq s_w^D$ if and only if $s^R \leq (1+n)k \leq s^D$. Under the dynamic efficiency condition, $r > 1+n$, if $s_w^R < s_w^D$, $s^R < rk$ holds, and if $s_w^R > s_w^D$, $s^D < rk$ holds. $X(1) = s^D + s^R - 2rk = 2((1+n) - r)k < 0$ holds by the market-clearing condition and the dynamic efficiency condition. When $s_w^R < s_w^D$, $X(\alpha)$ is linearly decreasing in $\alpha$ because $s^R < rk$.

(i) If $s^D \leq rk$, $X(0) \leq 0$ holds. Thus, $X(\alpha) < 0$ for all $\alpha \in (0,1)$. $X(\alpha) \frac{\alpha^R - \alpha^D}{\Gamma}$ is negative.

(ii) If $s^D > rk$, $X(0) > 0$ holds. By $X(1) < 0$ and the fact that $X(\alpha)$ is linearly decreasing, we obtain that there exists a threshold of $\alpha$, $\alpha^D$, such that the effect is positive if $\alpha < \alpha^D$ and negative otherwise.

(iii) When $s_w^R > s_w^D$, whether $s^R < rk$ or $s^R > rk$ is indeterminate. Thus, whether $X(\alpha)$ is linearly increasing or decreasing in $\alpha$ is not determined. However, in this case, $X(0) < 0$ because $s^D < rk$. By combining $X(0) < 0$ with $X(1) < 0$, we obtain that $X(\alpha) < 0$ for all $\alpha \in (0,1)$. Thus, $X(\alpha) \frac{\alpha^R - \alpha^D}{\Gamma}$ is positive. □

C The proof of Lemma 3

The sign of the recipient’s indirect effect, $(s^R - rk) \frac{\alpha^R - \alpha^D}{\Gamma}$, depends on the signs of $(s^R - rk)$ and $(s_w^R - s_w^D)$, because, like the proof of Lemma 2, $s_w^R \leq s_w^D$ if and only if $\frac{\alpha^R - \alpha^D}{\Gamma} \geq 0$ and $s^R \leq (1+n)k \leq s^D$. Under dynamic efficiency, if $s_w^R < s_w^D$, $s^R < rk$ holds, and if $s_w^R > s_w^D$, $s^D < rk$ holds. If $s_w^R < s_w^D$, the
recipient’s indirect effect is negative because \( s^R < rk \). On the other hand, when \( s^R > s^D_w \), the sign of the recipient’s indirect effect corresponds with that of \( (s^R - rk) \).  

D The proof of Proposition 1

Denote \( \Lambda^D \equiv (s^D - rk)\frac{s^D - D}{r} - 1 \) and \( \Lambda^R \equiv (s^R - rk)\frac{s^R - D}{r} + 1 \). (19) and (20) are rewritten as follows:

\[
\frac{dV^D}{dT} = \Lambda^D + \alpha \Lambda^R, \\
\frac{dV^R}{dT} = \Lambda^R. 
\]  

Note that \( \Lambda^D = \frac{dV^D}{dT} \bigg|_{\alpha=0} \). Suppose \( \Lambda^D < 0 \) and \( \Lambda^R > 0 \). By (A.1), \( \frac{dV^D}{dT} \) is linearly increasing with \( \alpha \). By substituting \( \alpha = 1 \) into (A.1) and noting that \( s^D + s^R < 2rk \) under dynamic efficiency, we obtain the following equation.

\[
\frac{dV^D}{dT} \bigg|_{\alpha=1} = (s^D + s^R - 2rk)\frac{R - s^D}{r} \leq 0 \quad \text{if and only if} \quad s^D \leq s^D_w. 
\]  

By (A.3), we obtain that when \( s^D_w < s^D \), \( \frac{dV^D}{dT} < 0 \) holds for all \( \alpha \in [0, 1) \). On the other hand, we obtain that when \( s^D_w > s^D \), \( \frac{dV^D}{dT} < 0 \) if \( \alpha < \alpha_1 \equiv -\frac{\Lambda^D}{\alpha^R} \) and \( \frac{dV^D}{dT} > 0 \) holds if \( \alpha > \alpha_1 \).  

E The proof of Proposition 2

All the cases that we examine in the general situation are denoted as follows: (a) \( (\Lambda^D, \Lambda^R) = (-, +) \); (b) \( (\Lambda^D, \Lambda^R) = (+, +) \); (c) \( (\Lambda^D, \Lambda^R) = (-, -) \); (d) \( (\Lambda^D, \Lambda^R) = (+, -) \). However, we can prove that Case (b) cannot occur when \( s^R_w < s^D_w \), and Case (c) cannot occur when \( s^R_w > s^D_w \). The proofs are by contradiction. Note that \( s^D + s^R < 2rk \) holds under the dynamically efficient condition. Suppose that \( \Lambda^D > 0 \) and \( \Lambda^R > 0 \) hold when \( s^R_w < s^D_w \). \( \Lambda^D > 0 \) and \( \Lambda^R > 0 \) hold if and only if \( (s^D - rk)\frac{s^R - D}{r} > 1 > -(s^R - rk)\frac{s^D - D}{r} \). Given \( \frac{s^D - D}{r} > 0 \), \( s^D - rk > -(s^R - rk) \). This contradicts \( s^D + s^R < 2rk \). Likewise, suppose that \( \Lambda^D < 0 \) and \( \Lambda^R < 0 \) when \( s^R_w > s^D_w \). \( \Lambda^D < 0 \) and \( \Lambda^R < 0 \) if and only if \( -(s^R - rk)\frac{s^D - D}{r} > 1 > (s^D - rk)\frac{s^R - D}{r} \). As \( \frac{s^D - D}{r} < 0 \), \( -(s^R - rk) < s^D - rk \). This also contradicts \( s^D + s^R < 2rk \).

Similarly to the proof of Proposition 1, (A.3) is satisfied under \( \alpha = 1 \) in all the cases. By the continuity of the function \( \frac{dV^D}{dT} \) with regard to \( \alpha \), it is satisfied that \( \frac{dV^D}{dT} \leq 0 \) if and only if \( s^R_w \leq s^D_w \) when \( \alpha \) is in
the neighborhood of unity.
References


